Order Splitting and Searching for a Counterparty

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Abstract. Large and patient traders have strong incentives to find natural counterparties, as it allows them to trade larger amounts at lower costs. The key challenge is that buyers and sellers must coordinate to find and trade with each other. Our theoretical model shows that searching for a counterparty can be done by means of order splitting, where a large quantity is broken up into many individual trades executed over the trading day. Order splitting facilitates the coordination, as it signals one's trading interest to the market and helps detecting the presence of counterparties. We confirm empirically that the presence of counterparties can be detected in real-time, and that the behavior of counterparties affects parent order characteristics. For example, for a one-standard deviation increase in volume by a natural counterparty, order sizes are approximately 17% larger and have an implementation shortfall 7.6 basis points lower, which is sizeable compared to a sample average of 6.2 basis points.

Key words: Order splitting, Signalling, Liquidity

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1. Introduction

Institutional investors typically trade large quantities of shares by splitting up the parent order into many small child trades, executing them gradually over the trading day. In this paper, we argue that order splitting may serve as a coordination mechanism, allowing institutional investors to strategically interact with the orders present in the market.

The intuition is as follows. Order splitting gradually reveals trading intentions to the market. If a natural counterparty is present, i.e., a trader with a different private value, then he will also signal his presence through order splitting. As both traders observe order flows and gradually infer each others' private values, they can start trading larger amounts without moving prices—realizing larger gains from trade. However, if all traders in the market have similar private values, the trading pressure will quickly move the price towards this value and profitable trading opportunities disappear.

We formalize this argument in model of a dynamic financial market. Two long-lived investors A and B receive a normally distributed private value shock for the asset at the start of the game, and trade each period to maximize expected gains from trade. Trading is modeled using the batch auction framework of Kyle (1985), where a market maker and a noise trader are also present.[1] We solve for the unique linear Bayesian Nash equilibrium, and find a recursive solution for the optimal trading strategies. In a given period, the optimal trade is linear in two state variables. First, each trader trades on his own private valuation to directly generate a gain from trade. Second, they trade on the expected private value of the counterparty with a negative parameter. That is, if the counterparty is expected to buy, then the current trader will sell more (or buy less) to provide liquidity.

The novel feature of the model is that each trader tries to signal his own private value and learn the private value of the other. In equilibrium, both traders provide liquidity to each other, and this liquidity provision gets stronger over time as they learn each others' private valuations more precisely. Further, as time passes, both traders anticipate more liquidity provision by the

[1] We deviate from Kyle with a strong assumption: we consider a private values environment and abstract away from asymmetric information and informed. The aim of the model is to focus on the strategic interactions between A and B, and we wish to minimize the role of the market maker.

other, which in turn allows them to trade more aggressively on their own private valuation. The two effects reinforce each other, which effectively is a form of coordination where both traders eventually trade very aggressively and realize large gains from trade. This coordination is reminiscent of pre-announcing trading interest or sunshine trading (Admati and Pfleiderer, 1991), but arises here in a dynamic context.

The signaling result is surprising since investors typically split up an order to camouflage their trading (as in, e.g., Kyle (1985) or Vayanos (1999)). By hiding their trades, an investor also reduces the risk of "predatory trading," where other investors exploit the order by trading in the same direction (Brunnermeier and Pedersen, 2005). Predatory trading is outside the scope of the model, but it would most likely not arise. Our investors have a private value for the asset, meaning they simply buy or sell if the price is below or above this value. Predatory trading only arises when a trader is forced to liquidate an exogenous position.

The model yields three key testable predictions which we take to the data. First, the sizes of parent orders are highly endogenous because they depend on the private value of the counterparty. That is, if one has a positive private value and the other a negative one, both will trade very large quantities. However, if both investors have positive values (or both negative), then the price will quickly adjust and aggregate quantities will remain small. This contrasts with the optimal execution literature which assumes an investor must trade a fixed (exogenous) amount before a deadline (see e.g., Almgren and Chriss (1999, 2000)). The second prediction is that each trader learns about the private value of the counterparty by analyzing price changes. This Bayesian updating gradually improves over time and affects the trading behavior. The third testable prediction is that the presence of counterparties strongly affects institutional trading costs. This predictions quantifies the advantage of searching and finding a natural counterparty.

We test the three predictions using a proprietary dataset of trades and quotes with trader identification for Finnish index stocks in 2007. We identify 30,092 institutional parent orders and their child trades, following the definition in the literature (Korajczyk and Murphy, 2018, van Kervel and Menkveld, 2018). The key advantage is that we observe all parent orders active at any given point in time, and can hence investigate how they interact with each other.

Parent orders are frequently overlapping. During the life of an average order order, 17.3% of market volume is executed by other parent orders in the opposite direction to the current order, and 8.2% in the same direction. Consistent with the model, parent orders are more likely to trade in opposite direction (i.e., one is buying and the other selling).

We confirm the first prediction that the institutional order direction and size depend on the presence (and size) of counterparties. A Logit regression reveals that a new parent order is 4% more likely to be a buy (rather than a sell) for each additional sell parent order active in the 30-minute interval prior to the start of the order. Similarly, a one standard deviation increase in sell volume by existing orders in this interval is correlated with an increase in the size of a new buy order of 15.7%. The results are symmetric for existing buy orders on new sell orders. These magnitudes are economically sizable, but we caution that they are only correlations.

The second prediction states that an investor can detect the presence of a counterparty by analyzing the price patterns. The model has Bayesian update equations, and empirically we propose to proxy these by using the realized spread of a parent order. The intuition is as follows. Suppose the current order is buying and pushing up the price, while another sell order is active pushing back the price. In this case, the selling pressure causes a reversal in prices which *increases* the buy orders' realized spread. If the other investor was buying this spread would decrease.[2]

Empirically, we confirm that, at the parent order level, the realized spread of an order is correlated with trading volume of other parent orders in the same and opposite direction, as predicted by the model. The economic magnitudes are sizable, proving that the realized spread may indeed serve as a detection mechanism.

We then continue to show that the realized spread also works as a detection mechanism *in real time*. Measured at the half-hour frequency we find that the realized spread in period $t-1$ predicts the order flow (participation rate) of the current parent order in period t. We also find that the realized spread is correlated with the participation rate of other institutional investors—both in the same and opposite direction of the current order. The signs of the correlations are

[2]In the traditional microstructure literature, the realized spread is a proxy for the profits to the market marker. This interpretation is no longer valid in our model, where all reversals in prices are due to the order flows of the two strategic traders—which is captured by the realized spread.

consistent with the predictions of the model. This suggests that the realized spread is in fact a measure that can be constructed in real time to detect a counterparty. Important to note is that any institutional investor can construct this measure, as it does not require our proprietary data.

The third and final prediction of the model relates the impact of a counterparty on trading costs. Regressions at the parent order level confirm that a one standard deviation increase in trading volume by institutional investors in the opposite direction is correlated with a reduction in the implementation shortfall of 7.6 basis points, which is sizable given a sample average of 6.2 basis points. We also find that the coefficient on institutional trading volume in the opposite direction is similar to the impact of one's own order size on implementation shortfall. In the literature, the largest determinant of implementation shortfall is the size of the order, and our analysis reveals that the impact of other institutional orders is of similar magnitude. This suggests that the potential advantage of having a natural counterparty is indeed quite large.

We then analyze the source of the trading cost advantage of counterparties. We find that a parent order is more likely to be liquidity motivated when other parent orders are trading in the opposite direction. That is, such orders have a low permanent price impact. Conversely, when multiple parent orders trade in the same direction, all have a much higher permanent price impact. This suggests that natural counterparties are particularly important for liquidity motivated traders, and this corresponds well to the private value environment in the theoretical model.

At this point we want to stress an important caveats. The empirical results are mere correlations, and we cannot interpret these as causal effects. In fact, obtaining causality is particularly challenging given that the theory shows that institutional orders highly endogenous: one order simultaneously affects and is affected by other orders. Despite this issue, we still believe the model is useful in guiding the empirical analysis and in interpreting the results. The fact that our results are consistent with many different predictions of the model suggests that the tradeoffs and mechanisms in the model are empirically at play as well.

The dynamic interaction between strategic investors has become increasingly important with the rise of order splitting algorithms and other dynamic trading strategies. There are two papers most closely related to our work. Vayanos (1999) studies dynamic interactions between multiple large traders who trade to hedge endowment shocks. They trade slowly to camouflage these shocks, as revealing this information would move prices against them. Slow trading prevents large price impacts, but reduces welfare because of lower risk sharing benefits. This result is opposite to our model, where the liquidity motivated traders want to signal their private valuations to coordinate liquidity provision. More recently, Choi, Larsen, and Seppi (2018b) study strategic interactions between multiple liquidity motivated traders in a continuous time setting. The traders must trade an exogenous amount, and their performance is benchmarked relative to a TWAP strategy. In equilibrium, the traders provide liquidity to each other which is traded off against higher penalties for deviating from the TWAP benchmark.

Our work is also related to papers using the Kyle (1985) framework to model interactions between strategic traders. Choi, Larsen, and Seppi (2018a) analyze the interaction between a long lived informed trader and a liquidity motivated trader. They show that the liquidity motivated trader, who must trade a fixed amount before a deadline, gradually learns about the fundamental value through his trading. Other papers focus on strategic interactions between multiple informed traders (e.g. Foster and Viswanathan (1996) and Back, Cao, and Willard (2000)).

2. Model

In this section we propose a model of a dynamic financial market where two strategic traders, A and B, have private values for an asset. We show that they have an incentive to signal their private value (i.e., their type) to realize larger gains from trades in future rounds. Our interpretation is that investors split up orders to find a counterparty in the market.

2.1 Setup

The market is populated by a market maker, two strategic traders, and a noise trader. The trading mechanism follows Kyle (1985), where trading occurs sequentially in a number of trading periods. Each period is an anonymous batch auction where the two strategic traders and a noise trader submit market orders. The market maker observes the aggregate (net) order flow, and clears the market by setting the price.

We deviate from Kyle (1985) with a very strong assumption: we abstract away from asymmetric information, and assume the market maker uses a linear pricing rule with an exogenous price impact parameter λ. Effectively, the market maker has a constant marginal cost λ of holding inventory, and she forgoes an optimization. There is no informed trading or adverse selection, and instead we focus on a model with private values. We simplify as much as possible the role of the market maker, and rather want to analyze the strategic interactions between two liquidity motivated traders.

The strategic traders $i = \{A, B\}$ are risk neutral, and have a private valuation (i.e., "type") for the asset that is centered around the public value v of the asset, $v + \tilde{\theta}_i$, with $\tilde{\theta}_i \sim \mathcal{N}(0, \Sigma 2_0)$. The public value does not change over time, and the initial price $p_0 = v$. This implies that if $\tilde{\theta}_A > 0$, trader A would be a natural buyer, and if $\tilde{\theta}_A < 0$ he would be a natural seller. Of course, depending on the realization of the price path, a trader may switch between buying and selling depending on $sign(v + \tilde{\theta}_i - p_n)$. Here we deviate from the optimal execution literature, which assumes a trader must trade an exogenous amount before a deadline (e.g., Almgren and Chriss, 1999). There are $n = 1, \ldots, N$ trading rounds where the strategic traders submit orders $x_{A,n}$ and $x_{B,n}$.

There is a noise trader who submits in each period a random order $\tilde{e}_n \sim \mathcal{N}(0, \sigma_e^2/N)$, which are all independent from each other and from the types of the strategic traders. The order flow y_n becomes

$$y_n = x_{A,n} + x_{B,n} + \tilde{e}_n. \tag{1}$$

The market maker sets the price p_n as a linear function of the order flow:

$$p_n = p_{n-1} + \lambda y_n. \tag{2}$$

In this setup, both traders are aware of each others presence, but they do not know each others type (i.e., the realization of $\tilde{\theta}$). Possibly, one is a natural buyer and the other a natural seller, meaning large gains of trade could be had. However, it is equally likely that both traders want to trade in the same direction (i.e., both are natural sellers or both natural buyers). In that case gains of trade will be lower, but are still positive since their types are continuous.

Each trader i is strategic and at period n maximizes the expected wealth over all the remaining periods:

$$Value_{i,n} = \max_{x_{i,n+1}} \mathbb{E}\left[\sum_{j=n+1}^{N} x_{i,j}(v + \widetilde{\theta}_i - p_j)|\widetilde{\theta}_i, y_1, \ldots, y_n\right]. \tag{3}$$

At time $n = 0$ a trader can only condition on his type, but in subsequent periods he can also condition on the past aggregate orders $y_1, \ldots, y_n$ when choosing order $x_{i,n+1}$.

Definition 1. *The equilibrium concept is a Bayesian Nash equilibrium, which is a set of functions such that:*

- *Given the optimal strategy $x_{B,n}$ by trader B, the trades $x_{A,n}$ maximize trader A's objective function in Equation 3.*
- *Given the optimal strategy $x_{A,n}$ by trader A, the trades $x_{B,n}$ maximize trader B's objective function Equation 3.*

Intuitively, we show that if one trader follows the equilibrium strategy, the other has no incentive to deviate. We construct a linear Bayesian equilibrium and conjecture that the optimal strategies take the form:

$$\begin{aligned} x_{A,n} &= \alpha_n(\tilde{\theta}_A - p_{n-1}) + \beta_n(e\theta_{B,n-1} - p_{n-1}), \qquad \beta_1 = 0, \\ x_{B,n} &= \alpha_n(\tilde{\theta}_B - p_{n-1}) + \beta_n(e\theta_{A,n-1} - p_{n-1}) \end{aligned} \tag{4}$$

We consider symmetric strategies (i.e., both traders use the same parameters α_n and β_n), and use $e\theta_{B,n}$ to denote trader A's expectation of $\tilde{\theta}_B$ after round n. Similarly, trader B's expectation of type A after round n is $e\theta_{A,n}$. The constants $\{\alpha_n, \beta_n\}_{n=1}^N$ are determined by maximizing Equation 3.

Crucially, the optimal strategy of trader A depends on his belief of type B ($e\theta_{B,n-1}$), but also on *his belief what B believes type A is.* This comes from the expected wealth Equation 3, which shows that trader A's trade in round n, $x_{A,n}$, depends on the expected price p_n, which in turn depends on the trade by investor B, $x_{B,n}$.

This second-order expectation will turn out to be a simple one because of the law of iterated expectations. To see this, note that the information sets of both traders at time n share a common component (the public information captured by order flow $y_1, \dots, y_{n-1}$), and a trader specific component (their own type $\widetilde{\theta}_i$). To form the dynamics of the process of $e\theta_{B,n}$, it will turn out useful to first track the public component. Denote the public process by

$$\begin{aligned} q_n :=& \mathbb{E}\left[\tilde{\theta}_A | y_1, \dots, y_n\right] = \mathbb{E}\left[\tilde{\theta}_B | y_1, \dots, y_n\right] \\ =& \mathbb{E}\left[\tilde{\theta}_A | q_{n-1}, y_n\right] \\ =& q_{n-1} + \delta_n (y_n - \mathbb{E}[y_n | q_{n-1}]), \qquad q_0 = 0. \end{aligned} \tag{5}$$

The first line shows that, using publicly observed order flow only, one cannot distinguish between $\tilde{\theta}_A$ and $\tilde{\theta}_B$, because both traders use symmetric strategies. The second line shows that the history of the order flow $y_1, \dots, y_n$ can be summarized by just q_{n-1} and y_n.

Now, trader A can form the expectation $e\theta_{B,n}$ by using q_n and his own type:

$$\begin{aligned} e\theta_{B,n} :=& \mathbb{E}\left[\tilde{\theta}_B | \tilde{\theta}_A, y_1, \dots, y_n\right] \\ =& \mathbb{E}\left[\tilde{\theta}_B | q_n\right] + \gamma_n (\tilde{\theta}_A - \mathbb{E}\left[\tilde{\theta}_A | q_n\right]), \\ =& q_n + \gamma_n (\tilde{\theta}_A - q_n), \qquad e\theta_{B,0} = 0. \end{aligned} \tag{6}$$

Trader B can make a symmetrical update, using the same parameter γ_n (this follows from the traders having identical variances of their private valuation, Σ_0):

$$\begin{aligned} e\theta_{A,n} :=& \mathbb{E}\left[\tilde{\theta}_A | \tilde{\theta}_B, y_1, \dots, y_n\right], \\ =& q_n + \gamma_n (\tilde{\theta}_B - q_n), \qquad e\theta_{A,0} = 0. \end{aligned} \tag{7}$$

The higher order expectation, i.e., A's belief on B's expectation of $\tilde{\theta}_A$, becomes just a simple update:

$$
\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[\tilde{\theta}_A|\tilde{\theta}_B, y_1, \ldots, y_n\right]|\tilde{\theta}_A, y_1, \ldots, y_n\right] &= \mathbb{E}\left[e\theta_{A,n}|\tilde{\theta}_A, q_n\right], \\
&= \mathbb{E}\left[q_n + \gamma_n(\tilde{\theta}_B - q_n)|\tilde{\theta}_A, q_n\right], \\
&= q_n + \gamma_n(e\theta_{B,n} - q_n).
\end{aligned} \tag{8}
$$

2.2 The optimal execution strategy

We first tackle the problem by trader A. By symmetry, we then have the optimal strategy for B as well.

The key challenge is that we are looking for equilibrium strategies, meaning we must allow for potential deviations off the equilibrium path (as in Foster and Viswanathan (1996), and, more recently, in Choi, Larsen, and Seppi (2018a)).[3] For example, if A deviates in round 2 by trading an additional unit, we must show that the equilibrium strategy in round 3 and onwards is still optimal. We want to stress that a deviation off the equilibrium path has two consequences. First, it changes the current (and future) state variables directly ($p_n, q_n, e\theta_{A,n}, e\theta_{B,n}$), which determine the beliefs by B and his trades in future rounds. Fortunately, the updating rules and strategies are all linear, meaning the future changes in the state variables by deviating are *fully* predictable to A.

Second, when deviating by choosing a different trade $x_{A,n}$, the information set of trader A does *not* change. That is, A can always filter his own trade from the order flow when forming expectations about $\tilde{\theta}_B$. Thus, the learning by A is the same on and off the equilibrium path. The challenge is that his beliefs are no longer given by Equations 5 and 6, but instead need to be tracked separately. It is for this reason that Foster and Viswanathan (1996) introduce "hat-processes" to track all state variables on the equilibrium path, along with the "regular" processes which may be affected by deviations off the equilibrium path. In the optimization, a deviation changes the realizations of the regular processes, but not the hat-processes as it capture the beliefs.

For our purpose, we only need to form a hat-process for A's belief of type B on the equilibrium

[3] Note that when searching for the optimal strategy of A, we only allow A to deviate: trader B is assumed to follow the equilibrium strategy (see definition 1). We do not allow both to deviate simultaneously.

path: $\widehat{e\theta}_{B,n}$.[4] To form this belief, note that A can always construct the unexpected or surprise component of the order flow, denoted by z_n^A:

$$\begin{aligned} z_n^A :=& y_n - \mathbb{E}\left[y_n|\tilde{\theta}_A, y_1, \ldots, y_{n-1}\right], \\ =& y_n - x_{A,n} - \alpha_n\left(\widehat{e\theta}_{B,n-1} - p_{n-1}\right) - \beta_n\left(q_{n-1} + \gamma_{n-1}(\widehat{e\theta}_{B,n-1} - q_{n-1}) - p_{n-1}\right). \end{aligned} \tag{9}$$

We take the order flow Equation 1, insert the strategy rule and higher order expectation (4 and 8), and replace $\tilde{\theta}_B$ by the expected value $\widehat{e\theta}_{B,n-1}$. For A, the process z_n^A is informationally equivalent to y_n. If A deviated in the past, he (predictably) changed the state variables q_{n-1} and p_{n-1}, which accordingly change the expected trade $x_{B,n}$ (represented by the last two terms in Equation 9). As such, z_n^A captures the unexpected component of the order flow, and allows him to form:

$$\begin{aligned} \widehat{e\theta}_{B,n} :=& \mathbb{E}\left[\tilde{\theta}_B|\tilde{\theta}_A, z_1^A, \ldots, z_n^A\right], \\ =& \widehat{e\theta}_{B,n-1} + r_n z_n^A, \qquad e\theta_{B,0} = 0. \end{aligned} \tag{10}$$

On the equilibrium path, $e\theta_{B,n} = \widehat{e\theta}_{B,n}$. We are now ready to state the main result.

Theorem 1. *A unique recursive linear Bayesian Nash equilibrium exists, where (i) the parameters α_n and β_n in the strategy function (4) maximize each traders objective function (3), and (ii) the parameters $\{\delta_n, \gamma_n, r_n\}$ are consistent with Bayesian updating rules. Specifically, for trader A we have the recursive equations:*

$$\begin{aligned} x_{A,n} =& \alpha_n(\tilde{\theta}_A - p_{n-1}) + \beta_n(e\theta_{B,n-1} - p_{n-1}), \\ q_n =& q_{n-1} + \delta_n(y_n - \mathbb{E}[y_n|q_{n-1}]), \qquad q_0 = 0, \\ e\theta_{B,n} =& q_n + \gamma_n(\tilde{\theta}_A - q_n), \qquad e\theta_{B,0} = 0, \\ \widehat{e\theta}_{B,n} =& \widehat{e\theta}_{B,n-1} + r_n z_n^A, \\ \Sigma_n :=& Var(\tilde{\theta}_A|y_1, \ldots, y_n) = Var(\tilde{\theta}_B|y_1, \ldots, y_n), \end{aligned} \tag{11}$$

where Σ_n is the variance of type $\tilde{\theta}_A$ (and $\tilde{\theta}_B$) conditional on the history of order flows. Given the

[4] The other state variables (p_n, q_n) are not needed for learning, and therefore we do not require to track them separately on the equilibrium path.

exogenous (initial) variance Σ_0, the constants solve the unique recursive system:

$$
\begin{aligned}
\alpha_n =& b_{1,n} + b_{3,n}\frac{\gamma_{n-1}}{1-\gamma_{n-1}},\\
\beta_n =& b_{2,n} - b_{3,n}\frac{\gamma_{n-1}}{1-\gamma_{n-1}},\\
\delta_n =& \frac{(\alpha_n+\beta_n\gamma_{n-1})(\Sigma_0-2(\Sigma_0-\Sigma_{n-1}))}{\sigma_e^2+2(\alpha_n+\beta_n\gamma_{n-1})^2(\Sigma_0-2(\Sigma_0-\Sigma_{n-1}))},\\
\gamma_n =& \frac{\Sigma_n-\Sigma_0}{\Sigma_n},\\
r_n =& \frac{(\alpha_n+\beta_n\gamma_{n-1})\Sigma_0(\Sigma_0-2\Sigma_{n-1})}{\sigma_e^2+2(\alpha_n+\beta_n\gamma_{n-1})^2\Sigma_0(\Sigma_0-2\Sigma_{n-1})-\sigma_e^2\Sigma_{n-1}},\\
\Sigma_n =& \Sigma_{n-1} - \frac{(\alpha_n+\beta_n\gamma_{n-1})^2(\Sigma_0-2(\Sigma_0-\Sigma_{n-1}))^2}{\sigma_e^2+2(\alpha_n+\beta_n\gamma_{n-1})^2(\Sigma_0-2(\Sigma_0-\Sigma_{n-1}))}.
\end{aligned}
\tag{12}
$$

The recursive constants $\{b_{1,n}, b_{2,n}, b_{3,n}\}_{n=1}^N$ are defined in the Appendix.

The proof consists of two parts, and follows the structure of Choi, Larsen, and Seppi (2018a). The details of the derivations are in the Appendix.

Proof Part 1: Bayesian parameters.

The parameters δ_n, γ_n, r_n, are coefficients identified using standard Bayesian updating rules. They are forward recursions, depending on the conditional variance Σ_n, which represents the learning about the variance of the type of each trader using public order flow information. We only need one conditional variance, as each trader can make an additional update using their type. The recursive form of Σ_n in (12) crucially depends on the term $(\alpha_n + \beta_n\gamma_{n-1})$, which captures the intuition that each trader trades on her own type (with parameter α_n), and on the expected type of the counterparty (with parameter β_n), about which they learned in the previous round with parameter γ_{n-1} (see Equation 6).

Proof Part 2: The strategy of trader A.

The parameters α_n and β_n are used by both traders. Trader A will choose trades $x_{A,n}$ to maximize the objective function (3) (taking as given α_n and β_n used by B). In turn, this will identify α_n and β_n using Equation (4). We identify parameters α_n and β_n through a backwards induction of the objective function (3).

The problem potentially has eight state variables: $p_n, q_n, e\theta_{A,n}, e\theta_{B,n}$, which can be tracked on and off the equilibrium path (see discussion in the previous section). It turns out that the value function of trader A in Equation (3) can be summarized by a set of only three composite state variables:

$$Y_n^{(1)} = \tilde{\theta}_A - p_n, \qquad Y_n^{(2)} = \widehat{e\theta}_{B,n} - p_n, \qquad Y_n^{(3)} = \widehat{e\theta}_{B,n} - q_n. \tag{13}$$

The first quantity captures the private valuation of A, the second is A's belief of B's private valuation, and the third is necessary in forming A's belief on B's expectation of $\tilde{\theta}_A$ (the higher order expectation in Equation 8).

We start at the last period N, and work our way backwards. Trader A's profit of the last trade N can be expressed as $x_{A,N}Y_N^{(1)}$, which equals the value function

$$\begin{aligned} Value_N^A =& \mathbb{E}\left[x_{A,N}Y_N^{(1)}|\tilde{\theta}_A, y_1, \ldots, y_{N-1}\right], \\ =& \mathbb{E}\left[x_{A,N}(Y_{N-1}^{(1)} - \lambda\left(z_N^A + x_{A,N} + (\alpha_N + \beta_N)Y_{N-1}^{(2)} + \beta_N(\gamma_{N-1} - 1)Y_{N-1}^{(3)}\right)\right. \\ & \left.|Y_{N-1}^{(1)}, Y_{N-1}^{(2)}, Y_{N-1}^{(3)}\right], \\ =& x_{A,N}(\lambda x_{A,N} + (Y_{N-1}^{(1)} + (\alpha_N + \beta_N)Y_{N-1}^{(2)} + \beta_N(\gamma_{N-1} - 1)Y_{N-1}^{(3)}). \end{aligned} \tag{14}$$

In line 2 we fill in the Markovian recursion of $Y_N^{(1)}$ (defined in Equation A.12 in the Appendix), the price and order flow equations (2 and 1), the linear strategy of B (4), and substitute in the unexpected order flow z_N^A (9). Crucially, the equation shows how the value function in period N depends on the state variables of period $N-1$ and the parameters. In fact, the three state variables are chosen such that the value function takes this recursive structure with Markovian dynamics.

It is easy to see that the value function is quadratic in $x_{A,N}$. We proceed by taking the first order condition to identify the optimal trade $x_{A,N}^*$:

$$x_{A,N}^* \quad = \quad \frac{Y_{N-1}^{(1)} - \lambda\left((\alpha_N + \beta_N)Y_{N-1}^{(2)} + \beta_N(\gamma_{N-1} - 1)Y_{N-1}^{(3)}\right)}{2\lambda}. \tag{15}$$

A quadratic value function yields optimal strategies that are linear in the three state variables, and we define constants $\{b_{1,N}, b_{2,N}, b_{3,N}\}$ such that

$$x^*_{A,N} \quad = \quad b_{1,N}Y^{(1)}_{N-1} + b_{2,N}Y^{(2)}_{N-1} + b_{3,N}Y^{(3)}_{N-1}. \tag{16}$$

In the appendix we show that the three constants exist for any period n, and that they can be easily identified using a backward recursion.

On the equilibrium path, $\widehat{e\theta}_{B,n} = e\theta_{B,n}$, such that the third state variable becomes a linear combination of the other two. Just two state variables are sufficient to describe the system,[5] allowing us to write

$$x^*_{A,N} \quad = \quad \left(b_{1,N} + b_{3,N}\frac{\gamma_{N-1}}{1-\gamma_{N-1}}\right)Y^{(1)}_{N-1} + \left(b_{2,N} - b_{3,N}\frac{\gamma_{N-1}}{1-\gamma_{N-1}}\right)Y^{(2)}_{N-1}. \tag{17}$$

This solution corresponds directly to the conjectured linear strategy of Equation. (4), and identifies α_N and β_N in Equation. (12).

We insert the optimal trade (16) into the value function (14), and obtain a function that is quadratic in the three state variables of period $N-1$. It further depends on the constants $b_{1,N}, b_{2,N}, b_{3,N}$ and parameters α_N, β_N and γ_{N-1}. Now, the value function $Value^A_{N-1}$ consists of two parts: the trade, $x_{A,N-1}Y^{(1)}_{N-1}$, and the time $N-1$ expectation of $Value^A_N$. We iterate the system backwards one period using the recursive equations for the three state variables (A.12 – A.13 in the Appendix). We again take the first order condition to obtain $x^*_{A,N-1}$, identify $b_{1,N-1}, b_{2,N-1}, b_{3,N-1}$, and subsequently α_{N-1} and β_{N-1}, and insert the optimal trade into the value function. We repeat the procedure until period 1.

In each period n, the system of equations (12) is a fifth-order polynomial in Σ_n. The correct solution is the fourth root (given by economical restrictions on the parameter space and the second order condition).

[5] Since $Y^{(2)}_n := \widehat{e\theta}_{B,n} - p_n = q_n + \gamma_n(\tilde{\theta}_A - q_n) - p_n$ and that $\widehat{e\theta}_{B,n}$ is a linear function of $\tilde{\theta}_A$ and q_n (see Equation 6), on the equilibrium path the third state variable becomes a linear combination of the first two: $Y^{(3)}_n = \frac{\gamma_n\left(Y^{(1)}_n - Y^{(2)}_n\right)}{1-\gamma_n}$.

2.3 Testable predictions

In this section we develop a set of model predictions that follow directly from Theorem 1. Later, we take these to the data.

We first introduce the concept of "natural countparties," which is the situation when one trader wants to buy and the other sell:

Definition 2. *Traders A and B are called "natural counterparties" in period n when $sign(\tilde{\theta}_A - p_n) \neq sign(\tilde{\theta}_B - p_n)$.*

The next lemma shows that a traders' aggregate quantity depends on the private value of the other trader, and that the (absolute) size increases when both traders are natural counterparties.

Lemma 1. ***Endogenous order sizes***

A larger realization of type B ($\tilde{\theta}_B$) reduces the cumulative net trading volume by A, $\sum_{n=1}^{N} x_{A,n}$, i.e., she buys less or sells more. Further, the trades by A and B have a multiplier effect: $\frac{\partial x_{A,n+1}}{\partial x_{B,n}} < 0, \forall n$, ceteris paribus. This multiplier effect is persisting, affecting trades in period $n+1$ and all remaining trading periods.

A unit increase in $\tilde{\theta}_B$ increases trade $x_{B,n}$ by α_n, ceteris paribus (Equation 4). The accompanied price change $\Delta p_n = \lambda\alpha_n$, and the change in A's expectation of $\tilde{\theta}_B$ is $\Delta e\theta_{B,n} = \gamma_n\delta_n\alpha_n$ (see Equations 5 and 6). This causes a change in A's trade in the next period of $\Delta x_{A,n+1} = -(\alpha_{n+1} + \beta_{n+1})\Delta p_n + \beta_{n+1}\Delta e\theta_{A,n} < 0$.[6] There is a multiplier effect, as the change $\Delta x_{A,n+1}$ reinforces the trades by both A and B in all subsequent rounds. These effects go through the same two channels: the changes in price and the expectation of the other's type. By symmetry, the effect of a change in $\tilde{\theta}_A$ on $x_{B,n}$ is identical. The multiplier effect leads to larger cumulative absolute trading volumes by both traders, and the absolute volumes increase in $|\tilde{\theta}_A - \tilde{\theta}_B|$, i.e., it is larger when both traders are natural counterparties.

Lemma 2. ***Detecting a counterparty***

An investor learns about the private value of the counterparty using the order flow process and

[6] The change in $\Delta x_{A,n+1} < 0$, which follows from the fact that $\beta_{n+1} < 0$ and $(\alpha_{n+1} + \beta_{n+1}) > 0$ for all n.

his own type, as stated by the Bayesian updating rule in Equation 6. Further, order flows and price changes are informationally equivalent, such that the Bayesian updating can be expressed in terms of price changes and trader type.

Intuitively, if trader A observes more buying pressure than anticipated, either trader B or the noise traders have been buying more. As a Bayesian, he thus revises his expectation of type B $e\theta_{B,n}$ upwards. Note that the order flow process is informationally equivalent to the price process since $p_n = p_{n-1} + \lambda y_n$.[7]

Lemma 3. ***Trading costs***
The per-share trading costs of A in period n, defined as $sign(x_{A,n})(p_n - p_0)$*, are lower when there is a natural counterparty, i.e., when the counterparty trades in opposite direction to the current order.*

We calculate trading costs by comparing the realized price to the price at start of the trading game p_0. Rather mechanically, if for example A is buying, then selling pressure by B reduces the price and accordingly A's trading costs.

2.4 Numerical example

In this section we offer a numerical example. Section A.2 in the Appendix describes a procedure to find the numerical solution.

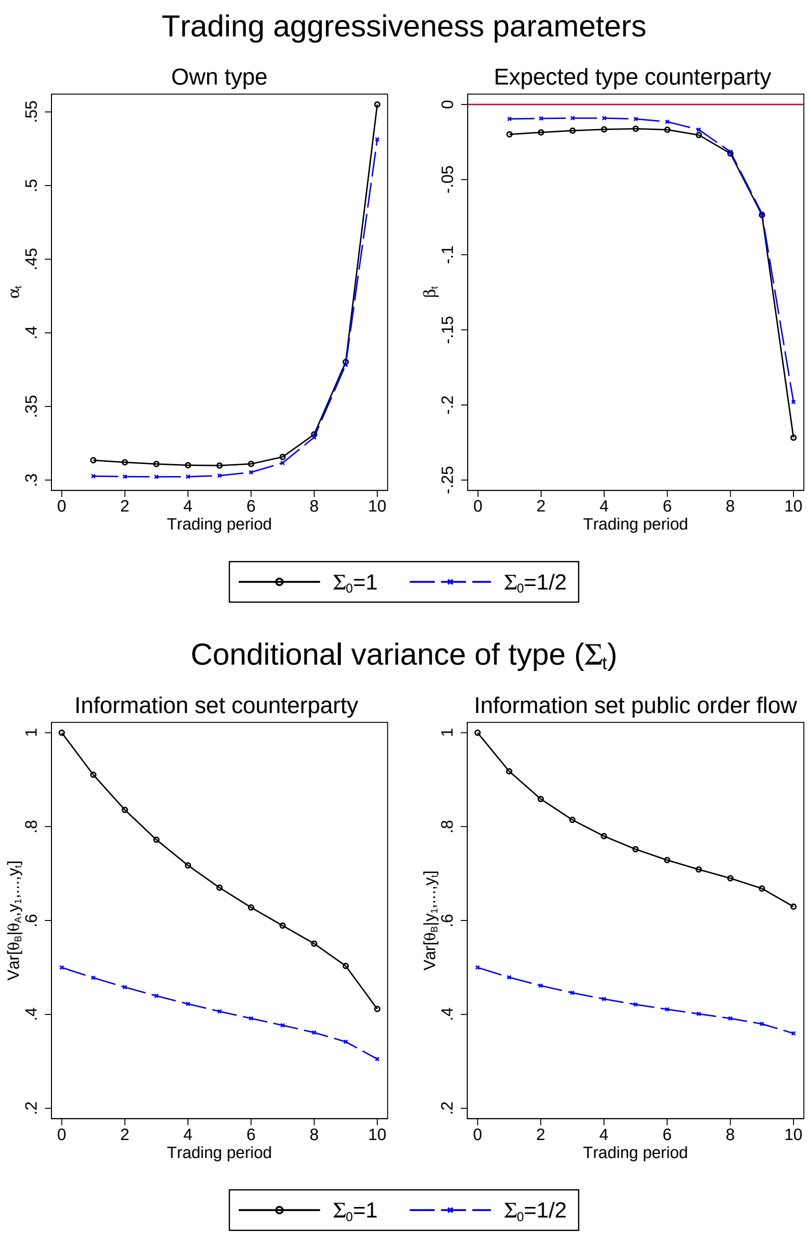
We first show the equilibrium parameters, which are all fixed before trading begins (but after the realization of the exogenous parameters, N, λ, σ_e^2, and Σ_0). We are also interested in the evolution of the state variables: the average quantities traded, the price, and the beliefs about the type of the counterparty. These are obtained via a simulation of the random variables in the game (i.e., the noise trades and the types of the two traders).

2.4.1 Model parameters Figure 1 shows the equilibrium parameters in a numerical example. We take a game of $N = 10$ trading rounds, and set the variance of noise trading each period to

[7] In the empirical analysis, we will use realized spreads to proxy for price changes in the price process.

Figure 1 Equilibrium strategies and learning about counterparties: Numerical example.

We use a numerical example to illustrate the equilibrium strategies and the conditional variance of the types. In Panel A, the parameter α_n represents the trading aggressiveness on ones own type $(\tilde{\theta}_A - p_{n-1})$, and parameter β_n on the expected type of the counterparty $(\mathbb{E}\left[\tilde{\theta}_B - p_{n-1}\right])$. Panel B shows the conditional variance of type B, using as information set public order flow and $\tilde{\theta}_A$ (left panel), or only public order flow (right panel). We use the following exogenous parameter values: $N = 10$ trading rounds, variance of noise trade per round is $\sigma_e^2 = 1$, and variance of private valuation $\sigma_\theta^2 = \Sigma_0 = 1$, exogenous price impact $\lambda = 1$. In each figure we plot the parameters for a lower variance of private valuation ($\Sigma_0 = 0.5$), and higher one ($\Sigma_0 = 1$).



one. We look at two cases: a low variance of the private valuations ($\Sigma_0 = 1/2$) and a high one ($\Sigma_0 = 1$). We learn the following.

- The parameters α_n represent the trading aggressiveness of A on his own private valuation $(\tilde{\theta}_A - p_{n-1})$. The top left panel shows a clear J-shape: α is approximately 0.32 in early periods, but increases to 0.55 in the last period. The explanation of the J-shape is as follows. By symmetry, trader B follows a strategy with identical parameters. Aggressive trading by B moves the price, which, in expectation, increases the magnitude of A's private valuation $(\tilde{\theta}_A - p_{n-1})$,[8] meaning A can trade more aggressively in turn.

- The parameters β_n represent the trading aggressiveness on the expected type of the counterparty B. This parameter is negative, meaning A trades in the opposite direction of the expected valuation of B. Effectively, A provides liquidity to B (i.e., pushing back the price pressure caused by B). This is a form of market making, which is profitable to A because the price pressure caused by B is independent of A's own private valuation. The β_n follow an inverted J-shape: the magnitudes increase over time as A gradually infers more precisely the private value of B, and can thus better predict to the trades by B (and provide liquidity).

- These two effects reinforce each other in equilibrium. That is, as time passes trader A expects B to trade more aggressively on B's valuation (through an increase in α_n) and more liquidity provision by B (through an increase β_n), which together allow A to trade more aggressively on his private valuation. In turn, B anticipates A to trade more aggressively, which allows him to trade more aggressively as well. Effectively, the two traders gradually coordinate their trading to maximize gains from trade.

- The bottom panel of figure 1 shows the conditional variance of the type of trader B. The right figure plots results using as information set only public order flow, and the left figure using both public order flow and the realization of type A, $\tilde{\theta}_A$. For the solid black lines, the conditional variance starts at one and for trader A it reduces fairly linearly to 0.41 in

[8] The private valuation of A is normally distributed, centered around p_0. By adding independent price pressures by B, the expected value of $|\tilde{\theta}_A - p_{n-1}|$ increases.

round 10. There is substantial learning. Using only public information (right panel) there is substantially less learning, as the conditional variance reduces to 0.63 in round 10. As is typical with learning about noisy variables, when the starting variance is lower, we see that traders learn less about each others types.[9]

2.4.2 State variables In this subsection we track the evolution of the state variables. We are interested in three particular cases: when the two traders are natural counterparties (i.e., a buyer and a seller); when one is a buyer and the other is neutral; and when both are buyers. To represent the three cases, we fix $\tilde{\theta}_A = 1$, and choose $\tilde{\theta}_B = \{-1, 0, 1\}$.[10] The initial price is 1,000 to which these private values are added. We further choose the same parameters as the game in Figure 1, and simulate 1,000 paths of the noise trades. As before, we consider two cases: $\Sigma_0 = 1$ and $\Sigma_0 = 1/2$.

Figure 2 shows the average cumulate quantities traded by the two traders. In the left panel, when $\tilde{\theta}_B = -1$, both traders are natural counterparties and we see that they each trade in a fairly linear fashion over time, accumulating 3.63 shares in round 10. In the middle panel, when trader B is neutral, we see that A trades only 2.09 shares (57% of the first case), while B sells 1.53 shares. Even though the private valuation of B is zero, the buying pressure by A quickly generates a profitable selling opportunity. In the right panel both traders are natural buyers, and each accumulate only 0.65 shares (18% of the first case).

The top panel of Figure 2 shows the equilibrium prices. In the left panel, when $\tilde{\theta}_B = -1$, the trading pressures of both parties cancel out, and the price remains flat, hovering around the starting price of 1,000. This variation is due to the price pressure of noise traders. In the middle and right panel the price quickly increases to the average of the two private valuations (1000.50 and 1001 respectively).[11] We see that the price eventually centers between the two

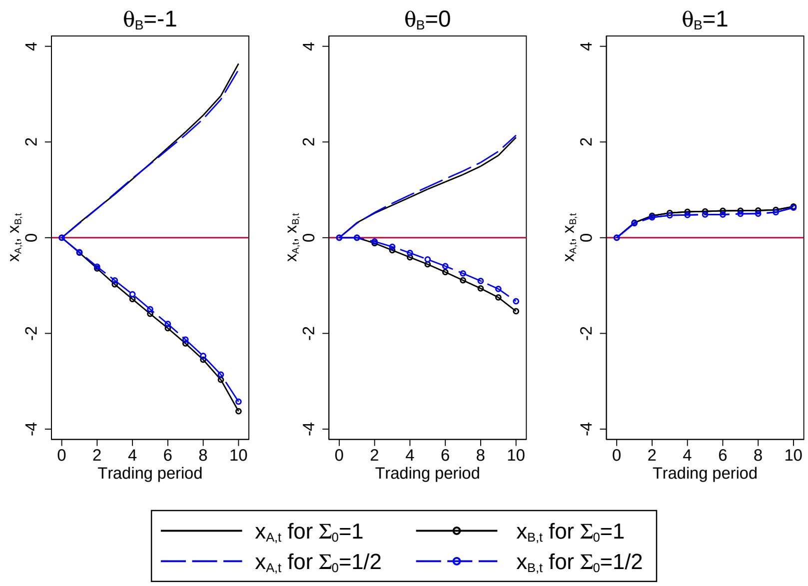
[9] A low initial variance implies that variation in order flow mostly represents noise trades, meaning it has a low signal to noise ratio and learning occurs only slowly.

[10] While we fix the realization of both types, the traders do not know this and play the game as if the types were drawn from a normal distribution.

[11] In the last periods of the right panel the price increases beyond 1001. While both traders have a private valuation of 1, their expectation of the counterparties' type is only 0.55 (see bottom panel). Therefore, with a price of 1001 they expect the counterparty to sell, and accordingly both buy a little more with parameter β_n. This effect is solely caused by fixing the types to equal one in this example; and it would not occur if we randomly draw the

Figure 2 Simulation: cumulative order flows.

We plot the average cumulative net order flow of both traders over time. Specifically, we fix the realization of $\tilde{\theta}_A = 1$, and vary $\tilde{\theta}_B = \{-1, 0, 1\}$ for the three panels. This setup reveals the three broad cases where trader B is a natural counterparty (left panel), is neutral (middle panel), or trades in the same direction (right panel). We use the parameter values of Figure 1, simulate 1,000 paths of noise trades, and take averages.



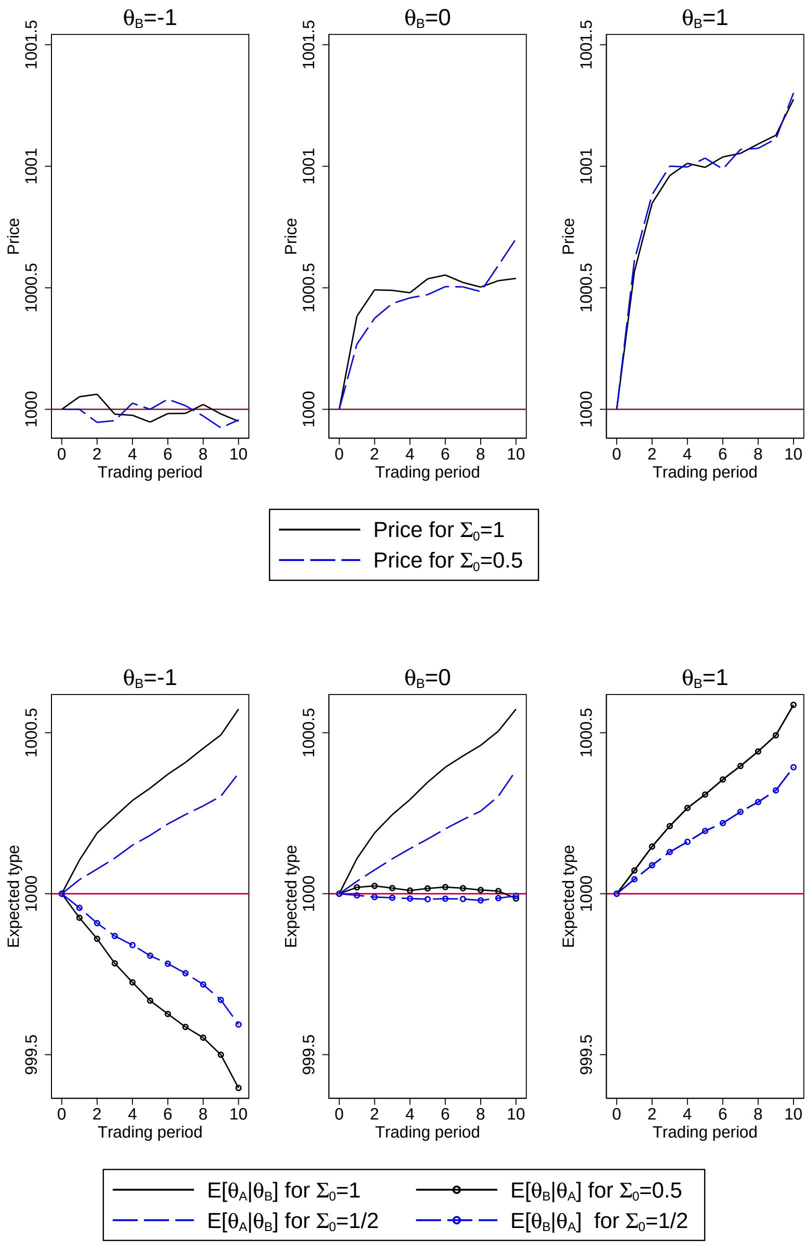
private valuations, at which point the buying price pressure of one gets canceled by the selling pressure of the other. We observe that the private valuation of the counterparty strongly affects a traders the overall trading volume.

The bottom of Figure 2 shows the beliefs the traders form about the type of their counterparty. Type A is fixed to $\tilde{\theta}_A = 1$, and in each panel trader B forms (almost) identical expectations about it. When $\Sigma_0 = 1$, the expectation $\mathbb{E}\left[\tilde{\theta}_A|\tilde{\theta}_B, y_1, \ldots, y_t\right]$ increases to 1000.58 in period 10, meaning that 0.58 of the 1 unit private valuation is learned. Trader A learns in a similar fashion about type $\tilde{\theta}_B$, which is -0.60 in panel 1, -0.015 in panel 2 and 0.58 in panel 3. The slight asymmetries are due to the sampling variance of noise trading, which shrink as the number of simulations increases.

types.

Figure 3 Simulation: price path and expected value counterparty.

We plot the average price path (top panel) and belief of the private valuation of the counterparty (bottom panel). We set the price p_0 to 1000 and fix the realization of $\tilde{\theta}_A = 1$ (such that her private valuation is 1,001). We consider three cases by varying $\tilde{\theta}_B = \{-1, 0, 1\}$. This setup reveals the three broad cases where the traders are natural counterparties (left panel), one trader is neutral (middle), or trading in the same direction (right panel). We use the parameter values of Figure 1, and average over 1,000 simulated paths of noise trades.



3. Empirical Analysis

In this section, we test the three key empirical predictions of the model. We wish to answer the following questions:

- How do institutional orders interact? What are the conditional probabilities of the direction of parent orders: do buys follow up sells (and vice versa)? How does trading pressure by existing parent orders affect the size of new parent orders?
- How can a trader find a counterparty, i.e., what is the detection mechanism? Can an investor analyze short-term price patterns to detect the presence of a counterparty in real time?
- What is the impact on trading costs as a result of strategic trading by counterparties? How large is the expected cost saving of having a counterparty?

Section 3.1 presents an overview of the institutional setting of the Nasdaq Helsinki market. Section 3.2 describes our data collection from public and proprietary sources, and provides summary statistics. Finally, Sections 3.3-3.5 address the three sets of questions raised above, respectively.

3.1 Institutional Setting

The investigated market segment for companies domiciled in Finland and listed on the Nasdaq Nordic Exchange [hereafter Nasdaq Helsinki] is similar to other European (and Asian) exchanges, where trading is conducted electronically in a central limit order book with no designated liquidity suppliers. Trading opens with an auction at 10.00 and closes at 18:30 after a relatively long post trading period that ends with a randomized closing auction. Off-market crossings are reported immediately for smaller trades, while larger blocks can be reported with a delay. During our sample period, the minimum tick-size is €0.01 for all sample stocks.

Since 2006, trading on the Nasdaq Helsinki market is pre-trade anonymous. However, broker dealers must report their trades to the central shareholder depository, which is administered by Euroclear Finland Ltd. Thus, all trades conducted on the Nasdaq Helsinki market segment for

Finnish stocks are reported to Euroclear, including trades in the upstairs market, dark pools, and those internalized within a brokerage firm.[12]

Despite its relatively small size, the Finnish stock market has become a significant part of the global portfolio. The Finnish stock market is home to well known companies in the technology sector, and their presence may also have alerted international investors to the other large companies at the exchange, typically in the engineering, forestry and resources industries. During our sample period, foreign investors held on average 61% of the market capitalization of the exchange, which is equivalent to approximately 235 billion Euro by the end of our sample period. 18% of Nokia, the largest capitalization company on the exchange, is held by 13f registered U.S. institutional investors during the period. Further, most of the approximately 200 common stocks listed during the period had foreign ownership of more than 1%. Hence, the results we draw from this dataset should have implications for our general understanding of financial markets, particularly in the context of institutional investors who operate globally.

3.2 Data

We obtain investor level transaction data for all Finnish listed firms from Euroclear Finland Ltd. for the period January 1, 2007 to December 31, 2007.[13] For each trade, the data contain the ISIN, date and time of trade, trade volume and price, and a shareholder account level identifier for the investor on both the active and passive sides of the transaction. These data are available until the end of 2007, after which the datasource contains transactions netted at the daily level.

We match the Euroclear data with tick-by-tick level transaction data provided by Nasdaq Helsinki and quote data obtained from Thomson Reuters Tick History at the millisecond level. Shares outstanding and adjusted closing price data are obtained from Standard and Poor's Compustat. We focus on transactions made during market open hours from 10:00 am to 18:30 pm.

[12]Overseas trading in American Depository Receipts is also be reflected in the depository but is subjected to extensive aggregation through nominee accounts.

[13]This dataset has also been used by Grinblatt and Keloharju (2000), Grinblatt and Keloharju (2001), Grinblatt, Keloharju, and Linnainmaa (2012), Linnainmaa and Saar (2012) and Berkman, Koch, and Westerholm (2014).

3.2.1 Identifying Parent Orders A parent order is a large order that is split up and executed as several child orders. We follow van Kervel and Menkveld (2018) and Korajczyk and Murphy (2018) to infer parent-order information from child trade executions, with several adjustments to account for the lower liquidity of Nasdaq Helsinki.

First, we identify potential parent orders by aggregating all child executions by a single institution for each stock and each day. Following the approach in the literature, a parent order extends over multiple days if there is a child trade in the last hour of one trading day and a child trade in the first half hour of the following trading day. Next, these potential parent orders are included in our final sample of parent orders if the following conditions are satisfied:

- *Large parent orders.* For stocks in the OMX Helsinki 25 index, we include orders that have an absolute net dollar volume above \$500,000, where the net dollar volume is the difference between the buy dollar volume and the sell dollar volume of the entire parent order. For stocks outside of the OMX Helsinki 25 index, we include orders that have an absolute net dollar volume above \$50,000.

- *High directionality.* We include orders with directionality above 0.9, where directionality is calculated as the absolute value of the difference between buying and selling dollar volumes, divided by the total volume for each parent order.

3.2.2 Summary Statistics for Orders and Transaction Costs Table 1 presents summary statistics for parent orders executed in Finnish stocks from January 1, 2007 to December 31, 2007. The average order is \$954,000 (Euros converted to USD at the sample average exchange rate), and lasts 0.44 trading days (or 3.7 hours, given 8.5 trading hours in a trading day). Further, 10.7% of our parent orders execute over multiple days. The average number of child trades per order is 34 but the median only 12. This suggests our sample consists of both parent orders that execute with a large number of small child trades, as well as orders that execute using a few large (block) trades.

Parent orders are executed fairly patiently, as the fraction of child trades traded actively (i.e., with a market order) is 40% on average (similar to van Kervel and Menkveld (2018) for the

Swedish market.) The orders have a 33.7% participation rate (parent order volume divided by single-counted market volume). This high number is partly caused by a subset of orders executed through a small number of larger block trades.

During the life of each order, we also track the volume executed by other institutional parent orders that trade in the same direction (same sign) and in opposite direction (opposite sign). We express these volumes as a fraction of (single counted) market volume. The average opposite sign is 17.3% of market volume and the average same sign only 8.2%. The difference between 17.3% and 8.2% suggests that other overlapping orders are much more likely to trade in opposite direction to the current order, meaning there is simultaneous buying and selling pressure. Further, the overlapping volume by other parent orders is substantial given an order's average participation rate of 33.7%.

We measure transaction costs with the implementation shortfall in basis points. It is defined as the percentage difference between the average price paid and the price at start of the parent order, and is multiplied by minus one for sell orders. The sample average implementation shortfall is 6.03 basis points which is fairly low but similar to van Kervel and Menkveld (2018) who find an average of 8.27 basis point for Swedish stocks. The order size weighted average is 36 basis points.

3.3 Lemma 1: Endogenous order sizes

The model predicts that the trading volume of one investor strongly depends on the trading by the other. For example, if the other investor is buying, the current investor is more likely to sell (and will sell larger quantities). Specifically, lemma 1 states that an investor's cumulative order size increases with the presence of a natural counterparty (idefined as a trader with opposite trading needs).

To test whether lemma 1 holds empirically we start with analyzing how the size and direction of a new order is correlated with the characteristics of existing orders and market conditions. For each new parent order, we take the 30 minutes interval prior to its start and track the buying and selling volumes of existing parent orders. For example, for a new parent buy order, we sum up the total child trade volume for existing buy orders (same sign) and for existing sell orders (opposite sign), in the 30 minutes prior to the start of the new parent order.

Next, we rank these volumes into terciles, separately for volumes that trade in the same direction and in opposite direction to the incoming parent order. To make a fair comparison, we use the same cutoff values for the terciles of the same sign and opposite sign volumes.[14] We add a category 0 if no volume is executed in the same or opposite direction.

Table 2 shows the frequency of new parent orders by the same sign and the opposite sign volume terciles. We report the percentage of total orders in parentheses (N=30,092). The table shows two main findings. First, comparing values of the upper triangle to the lower triangle, a new order is generally much more likely to trade in opposite direction of existing orders. For example, there are 2,636 cases where there is no volume in the same direction and a low-tercile volume in the opposite direction. This is significantly higher than the 1,888 cases where there is no volume in the opposite direction and a low-tercile level of volume in the same direction. Similarly there are 1,883 cases where there is no volume in the same direction and a high-tercile volume in the opposite direction, compared to 1,594 cases where there is no volume in the opposite direction and a high-tercile level of volume in the same direction. This suggests that buying pressure from existing orders attracts new selling orders (and vice versa), consistent with lemma 1 which states that institutional order sizes are endogenous and reinforce each other.

Second, we see that large trading volumes in both directions are highly clustered. For example, there are 1,104 cases with high same-sign and high opposite-sign volume, which is substantially more than the 627 cases where there was a low same-sign and opposite-sign volume. The clustering of high volumes is also consistent with lemma 1.

In a complementary analysis of order interaction, we investigate how the sign and log volume of a new order are correlated with the characteristics of existing orders. The aim is to confirm the results of Table 2 in a regression framework where we control for market conditions. We calculate all independent variables in the 30 minute interval prior to order start. The 30 minute window captures relatively recent market activity, given a typical order duration of three hours, which is important in the model where a current trade affects the counterparty's trade in the next period.

[14] The cutoff values are 2,226 shares between tercile 1 and 2, and 10,000 shares between tercile 2 and 3.

Table 3 presents the results. Columns 1 and 2 show logit regressions, where the dependent variable is equal to 1 for a new parent buy order and 0 for a new parent sell order. We include only parent orders lasting for more than 30 minutes to fully execute to keep a consistent sample. We present the marginal effects based on the means of the independent variables. In column 1 we see that for each additional existing sell order, the probability of a new buy order increases by 4.1% (second row), which is economically sizable and statistically significant at the 10 percent level. The impact of existing buy orders is -0.014 (first row) and insignificant. In column 2, for each additional standard deviation increase in log sell volume by existing orders, the probability of a new buy order increases by $2.7\% = 0.007 * 3.84$ points.[15] The magnitude of log buy volume is virtually the same with a coefficient of 0.008.

The regressions further show that a positive return in the half-hour prior to order start diminishes the probability of a buy order, which is consistent with investors strategically timing the start of the order (see e.g., Hendershott, Jones, and Menkveld (2013)). Market volume and volatility do not affect the direction of the new order, as expected.

For the remaining columns, we use the logarithm of buy or sell volume of the new parent order as dependent variable. We only use the volume executed during the first 30 minutes of the order's life, to be consistent across parent orders and limit look-ahead bias. We see that each additional existing sell order increases the volume of buy orders by 22.4% (column 3). Similarly, each additional buy order increases sell volume by 18.3% (column 5). We obtain similar results when we analyze the association between log volume of the new order and log of active order buy and sell volumes. A one standard deviation increase in existing sell order volume increases new buy volume by $15.7\% = 3.84 \times 0.041$ (column 4). The effect of existing buy volume on new sell volume is of similar magnitude, with a coefficient of 0.034 (column 6).

These are economically large effects, and we want to stress that we control for market volume in the regressions. Thus, the impact of existing parent orders on new order size does not simply go through an increase in market volume. Rather, institutional order sizes seem endogenous and reinforce each other, as predicted by the model.

[15] The standard deviation of log active sell volume is large with 3.84, which stems from both outliers and many observations where sell volume by existing institutional orders is zero.

At this point, we want to stress an important caveat. The empirical results are mere correlations, and we cannot interpret these as causal effects. In fact, obtaining causality is particularly challenging given that the theory shows institutional orders are highly endogenous. Despite this issue, we do believe we have shown that the empirical relations are consistent with the model.

3.4 Lemma 2: Order detection

We have shown that existing orders affect the direction and size of new orders. A much more specific prediction of the model is that an institutional investor can detect the presence of a counterparty by analyzing price patterns in the data. Specifically, lemma 2, states "An investor learns about the private value of the counterparty using the order flow process and his own type, as stated by the Bayesian updating rule in Equation 6." The update rule can be easily expressed in terms of price changes and trader type, since price changes depend linearly on order flows.

The intuition of detecting a counterparty is that if trader A observes more buying pressure than anticipated (i.e., conditional on his own trades), either trader B or the noise traders have been buying more. As a Bayesian, A thus revises upwards the expectation of the private value of B. The converse holds if there is more selling pressure than anticipated.

Finding an empirical proxy for this learning process is challenging for two reasons. First, learning occurs sequentially in the model—traders update after each trading period—but empirically, it is not clear what constitutes a trading period. Second, the parameters with which traders update change over the lifetime of the order. We therefore take a different approach, and argue that an investor can learn about the presence of counterparties by analyzing the *realized spread* of trades. The realized spread is a proxy for the reversal of prices and similar to the resiliency of the market. Intuitively, the price pressure of a buyer gets reduced when a large seller is active, i.e., the price reverts and the realized spread increases. Conversely, if both traders are buying, the price pressures reinforce each other and the realized spread will turn out lower.

The definition of the realized spread makes clear how it is affected by an institutional counterparty:

$$RS_i = D_i \frac{1}{K_i} \left(\sum_{k}^{K_i} Ln(P_k) - Ln(M_{k+\tau}) \right), \tag{18}$$

where $D_i = \{-1, 1\}$ if parent order i is a sell or buy order, respectively. The realized spread of a trade is the difference between the price P_k and the future midpoint $M_{k+\tau}$ taken τ trades later. Indeed, aggressive trading by counterparties affects this future midpoint $M_{k+\tau}$, and therefore the realized spread. We measure the realized spread at the parent order level by averaging over all $k = 1, \ldots, K_i$ trades executed on Nasdaq Helsinki during the lifetime of the parent order.

One remark is in order. The realized spread is typically used as a proxy for the profits to market markers.[16] This interpretation is no longer valid in our model. The market maker directly earns the instantaneous price impact (λ), but does not face adverse selection from informed traders. The reversals in prices *across trading periods* are caused by the order flows of the two strategic traders. The proposed realized spread captures these reversals.

We tackle the question of whether a counterparty can be detected from the price process with two analyses. In the first analysis, at the parent order level, we show that the realized spread indeed correlates with the trading volume by other institutional investors in both the same and opposite direction. We express these volumes as a fraction of overal volume in the market, as detecting a counterparty presumably is more difficult when there is lots of trading by non-institutional investors. This test exploits our rich dataset in which we observe the parent orders of many institutional investors with exact information on when they traded. An important benefit of using the realized spread, is that the realized spread is a statistic that can be constructed by any investor, i.e., it does not require any proprietary data. In the second analysis, we show that the realized spread measure can also identify counterparties in real time. Each parent order is split up into 30-minute intervals, and we show that the realized spread in interval t predicts institutional order flows by counterparties in interval $t + 1$. As such, we argue that the realized spread can serve as a real-time detection mechanism.

[16] Traditionally, the effective spread is decomposed into a price impact and realized spread. The effective spread is the instantaneous reward for the liquidity supplier, which, after subtracting the orders price impact to correct for adverse selection, represents the liquidity supplier's gross profit.

3.4.1 Order detection and the realized spread

Table 4 presents the results of a regression of the realized spread during a parent order on order characteristics. The realized spread is measured using mid-point prices 5, 10 or 20 trades from the original trade, and is averaged over all trades to obtain a variable at the parent order level. Generally the results show that *Opposite sign %*, the fraction of market volume traded by other parent orders in opposite direction, is positively and significantly correlated with the realized spread, and increasingly so with larger horizons for the mid-point benchmark. This supports our model prediction that a stronger (weaker) reversal of prices for A correlates with more selling (buying) pressure by B.

The economic magnitudes are as follows. A one-standard deviation increase in opposite sign % (25.4) increases the 5-trade realized spread from a sample average of 9.8 basis points to 10.6 basis points ($25.4 \times 0.033 = 0.84$ basis points). The effect becomes substantially larger once we look at the midpoint further in the future. A one-standard deviation increase in opposite sign % increases the 10-trade realized spread by $25.4 \times 0.092 = 2.3$ basis points, and the 20 trade realized spread by 4.6 basis points.

The magnitudes are similar for same-sign % in all three specifications (although the sign switches as expected). In fact, t-tests do not reject the null that the magnitude is equal in any of the specifications.[17] This implies that if the realized spread is used to detect the presence of counterparties, it will pick up the net effect of other institutional investors (i.e., same sign minus opposite sign order flow).

The coefficents of the other variables are as expected. Participation rate is insignificant in all three specifications. Ln trade size is negative (or insignificant), suggesting that splitting up an order into many small child trades (instead of a few large blocks) is correlated with a lower realized spread. Overall, the strongest predictor of the realized spread is the % of the order executed with market orders. This effect is mechanical however, as an executed limit order has a negative realized spread from the institutional traders point of view.

[17] Specifically, we test whether same-sign %$= -1\times$ opposite sign %. The p-values are 0.80, 0.55, 0.26 for the three specifications, respectively).

3.4.2 Order detection in real time We have demonstrated that the realized spread is correlated with the order flow by other institutional investors in the same and opposite direction at the parent order level. However, if the realized spread is to be used as a detection device, we need to dig deeper and show that it can predict other institutional flows *in real time*. To this end, we split up parent orders into half-hour intervals and create a panel dataset. We then measure all variables, including the realized spread, at the half-hour frequency. We are interested in estimating how the realized spread predicts future participation rate %, same sign %, and opposite sign %.

Table 5 shows two sets of results. Panel A contains predictive regressions, using only lagged values of all independent variables. Panel B shows the same regression but using contemporaneous values. Given the highly endogenous nature of all variables, we offer both panels for completeness. The theory predicts that the presence of a natural counterparty (i.e., more opposite-sign flow) (i) increases the realized spread, which (ii) in turn causes the investor to trade more. We confirm both effects. In panel A, we confirm that a higher realized spread in period $t-1$ positively predicts participation rate (coefficient of 0.044, column 1). A one standard deviation increase in the realized spread after 10 trades (34.69 at the half-hour frequency) increases the participation rate by $0.044 * 34.69 = 1.52\%$ relative to a sample average of 33.7%. In panel B we see that the realized spread is contemporaneously correlated with opposite sign %, with a slightly larger coefficient of 0.056. These magnitudes do not seem large, but our measure is only a crude proxy which ignores many other variables that are potentially informative of the presence of the counterparties.[18]

In Panel A column 1, we also see that lagged opposite sign % has a direct positive correlation with contemporaneous participation rate. This effect holds after controlling for the realized spread, and is directly consistent with Lemma 1 that order sizes of natural buyers and sellers reinforce each other. Panel A further shows that all dependent variables are highly autocorrelated, with AR(1) coefficients ranging between 0.33 and 0.467 across columns. This is a direct consequence of the use of order splitting algorithms. Finally we observe that the coefficients on the realized spread are of similar sign and magnitude in both panels. This again confirms that all variables

[18]Other candidate variables, for example, are the quoted liquidity on the bid and ask side in the order book, or particular patterns in the size, number and frequency of new limit and market orders.

are simultaneously determined. For this reason we do not estimate a VAR model with impulse response functions, as cause and effect cannot be distinguished.

In summary, the estimations confirm lemma 1 that order sizes are endogenous, through the positive correlation between *Opposite sign%* and *Participation rate %*, and lemma 2 that the realized spread is a detection device (i.e., correlated with *Opposite sign%*), which in turn positively predicts future traded quantities.

3.5 Lemma 3: Counterparties and trading costs

Lemma 3 predicts that trading costs are lowest when the two traders are natural counterparties (i.e., one is a buyer and the other a seller). In this section we first investigate the impact of overlapping orders on trading costs. We then analyze whether overlapping orders are more information motivated or more liquidity motivated. This reveals the trading motives of the institutional investors and is the source of the trading cost advantage.

3.5.1 Implementation shortfall and order flow Column 1 in Table 6 reports the estimation results for the correlation between implementation shortfall (IS) and market conditions and order characteristics. The IS is defined as the difference in basis points between the average price paid and the price at start of the parent order, and is multiplied by minus one for sell orders.

The variable *Opposite sign %* is negatively and significantly related to implementation shortfall with a coefficient of -0.300. A one-standard deviation increase in the variable reduces the IS by $25.4 * 0.357 = 7.6$ basis points (relative to a sample average of 6.03 basis points, Table 1). Furthermore, the magnitude of the coefficient on *Same sign%* is very similar with 0.363. This suggests that trading by other institutional investors affects trading costs linearly, and that the relevant statistic is the net volume (i.e., same sign % minus opposite sign %).

Importantly, these coefficient are virtually identical to that of an orders own participation rate (0.363). This suggests that trading by other institutions has a similar impact on implementation shortfall as ones' own order size. The literature typically finds that the main determinant of institutional trading cost is order size, and the current results show that the trades by other institutional investors have an effect of similar magnitude.

The remaining coefficients carry the expected signs. Order size, volatility, and the percentage of market order executions are all positively related to IS. The r^2 of this equation is 9.3% which is typical in the literature.

3.5.2 Overall, Permanent, and Transitory price impacts The model abstracts away from asymmetric information and informed trading, but empirically we cannot ignore these effects. For this reason, we use the approach of van Kervel and Menkveld (2018) to decompose an order's Overall Price Impact into a Permanent Price Impact and Transitory Price Impact. Specifically, Overall Price Impact (OPI) is the market adjusted stock return based on mid-quote prices from the beginning of the parent order to the end of the parent order, Permanent Price Impact (PPI) is the market adjusted stock return based on mid-quote prices from the beginning of the parent order to the mid-quote price 24 hours after the end of the parent order, and Temporary Price Impact (TPI) is the difference between OPI and PPI. We are interested in how these variables are affected by Same sign % and Opposite sign %.

The results in columns 2 to 4 in Table 6 further support our model predictions that the Overall Price Impact is lower with an increased presence of *Opposite sign%* orders, obtaining a coefficient of -0.426. The economic magnitude is sizable with $-0.426 \times 25.4 = 10.82$ basis points per standard deviation shock, relative to the sample average OPI of 4.6 basis points. The OPI is similar in nature to the implementation shortfall (column 1), which had a coefficient of -0.3. Here too the impact of Same sign % is positive and a bit larger in magnitude (coefficient of 0.597).

The decomposition of OPI into a permanent and transitory component reveals an interesting pattern: Same sign % increases PPI (0.806, column 3) while it decreases TPI (-0.21, column 4). This suggests that when multiple institutions trade in the same direction, they are likely to trade on fundamental information (i.e., a high permanent price impact and a small (or negative) reversal). The impact of Same sign % on PPI is twice as large as that of an orders own participation rate (0.367). This means that the expected PPI of two investors trading 10% of market volume each is $11.67 = 10 * .367 + 10 * .806$ basis points, which is larger than a single investor trading 20%

of market volume with a PPI of $7.34 = 20 * .367$ basis points. The reverse holds when Opposite sign % increases, suggesting that when two institutional investors trade in opposite directions, both are more likely to trade for liquidity reasons. This latter setting corresponds best to the theoretical model, and explains why trading costs are lower when having counterparty: both orders appear more likely to be liquidity motivated.

4. Conclusions

The dynamic interaction between strategic investors has become increasingly important with the rise of order splitting algorithms and other dynamic trading strategies. We present a model where two liquidity motivated traders signal and learn about each others' private values in order to maximize gains from trade. They effectively coordinate and accommodate each others' orders, in a process where trades by one push back the price pressure caused by the other.

As markets evolve and complex execution strategies are automated and executed faster, there is a need for more work to provide guidelines on how institutional investors should adapt. Using account level transaction data we show that order splitting and patiently waiting for a counterparty can be an effective way to manage transaction costs. We also show how publicly available data can be used to detect order flow in opposite direction to facilitate liquidity driven trading.

A. Appendix

A.1 Proof Theorem (1)

We prove the theorem in two parts, following the structure in the main text.

A.1.1 Kalman filter In this section we define the Bayesian updating parameters. To this end, it is useful to first introduce z_n, which is the unexpected order flow for an investor with only access to public information (e.g., the market maker). Note the difference with z_n^A, which conditions on

$\tilde{\theta}_A$ as well.

$$
\begin{aligned}
z_n :=& y_n - \mathbb{E}\left[y_n | y_1, \ldots, y_{n-1}\right], \\
=& y_n - \mathbb{E}\left[x_{A,n} + x_{B,n} | y_1, \ldots, y_{n-1}\right], \\
=& y_n - \mathbb{E}\Big[\alpha_n(\tilde{\theta}_A - p_{n-1}) + \beta_n(e\theta_{B,n-1} - p_{n-1}) + \\
& \quad \alpha_n(\tilde{\theta}_B - p_{n-1}) + \beta_n(e\theta_{A,n-1} - p_{n-1}) | y_1, \ldots, y_{n-1}\Big], \\
=& y_n - 2(q_{n-1} - p_{n-1})(\alpha_n + \beta_n), \\
=& e_n + (\alpha_n + \beta_n\gamma_{n-1})(\tilde{\theta}_A + \tilde{\theta}_B - 2q_{n-1}).
\end{aligned}
\tag{A.1}
$$

In the third line we insert the strategies used by the two traders. In the fourth line we recognize that q_n represents the expectation, using public information only, of all terms $\tilde{\theta}_A, \tilde{\theta}_B, e\theta_{B,n-1}$, and $e\theta_{B,n-1}$. The last line fills in the order flow equation.

The variables $z_1, \ldots, z_n$ are mutually independent, and the history of $z_1, \ldots, z_n$ is informationally equivalent to observing the history of $y_1, \ldots, y_n$. We can easily obtain the conditional variance

$$
Var\left[z_n | z_1, \ldots, z_{n-1}\right] = \sigma_e^2 + (\alpha_n + \beta_n\gamma_{n-1})^2\left(2\Sigma_{n-1} + 2(\Sigma_{n-1} - \Sigma_0)\right). \tag{A.2}
$$

The last term in brackets, $(2\Sigma_{n-1} + 2(\Sigma_{n-1} - \Sigma_0))$, equals the conditional variance of $\tilde{\theta}_A$ and $\tilde{\theta}_B$ plus two times the (conditional) covariance. We can now denote the conditional variance Σ_n by:

$$
\begin{aligned}
\Sigma_n :=& Var(\tilde{\theta}_A | y_1, \ldots, y_n) = Var(\tilde{\theta}_B | y_1, \ldots, y_n), \\
=& Var(\tilde{\theta}_A | z_1, \ldots, z_n), \\
=& \Sigma_{n-1} - \delta_n^2 Var(z_n), \\
=& \Sigma_{n-1} - \frac{Cov(z_n, \tilde{\theta}_A)^2}{Var(z_n)}, \\
=& \Sigma_{n-1} - \frac{(\alpha_n + \beta_n\gamma_{n-1})^2(\Sigma_{n-1} + (\Sigma_{n-1} - \Sigma_0))^2}{\sigma_e^2 + (\alpha_n + \beta_n\gamma_{n-1})^2\left(2\Sigma_{n-1} + 2(\Sigma_{n-1} - \Sigma_0)\right)}.
\end{aligned}
\tag{A.3}
$$

To explicitly show that Σ_n is a forward recursion, depending on the solutions to α and β, we can rewrite it as:

$$\Sigma_n \quad = \quad \Sigma_0 \frac{\sigma_e^2 + \Sigma_0 \sum_{i=1}^{n}(\alpha_i + \beta_i \gamma_{i-1})^2}{\sigma_e^2 + 2\Sigma_0 \sum_{i=1}^{n}(\alpha_i + \beta_i \gamma_{i-1})^2} \qquad with \quad \gamma_0 = 0. \tag{A.4}$$

For the conditional mean of $\tilde{\theta}_A$ using public information, denoted q_n, we have the parameter δ_n that solves the Bayesian update:

$$\begin{aligned}
q_n \equiv \mathbb{E}\left[\tilde{\theta}_A | z_1, \ldots, z_n\right] &= \mathbb{E}\left[\tilde{\theta}_A | z_1, \ldots, z_{n-1}\right] + z_n \delta_n, \\
&= q_{n-1} + z_n \delta_n, \qquad where \\
\delta_n &= \frac{Cov(z_n, \tilde{\theta}_A | z_1, \ldots, z_{n-1})}{Var(z_n | z_1, \ldots, z_{n-1})} \\
\delta_n &= \frac{(\alpha_n + \beta_n \gamma_{n-1})(\Sigma_{n-1} + (\Sigma_{n-1} - \Sigma_0))}{\sigma_e^2 + (\alpha_n + \beta_n \gamma_{n-1})^2 \left(2\Sigma_{n-1} + 2(\Sigma_{n-1} - \Sigma_0)\right)}.
\end{aligned} \tag{A.5}$$

In the second line we use the definition of q_n from Equation 5, and the fact that z_n is independent from previous realizations. The third line defines coefficient δ_n, which we rewrite in the last equation. Note the similarity with the conditional variance of Equation A.3.

In equilibrium, the recursion for γ_n is the parameter that solves the Bayesian update:

$$\begin{aligned}
e\theta_{B,n} \equiv \mathbb{E}\left[\tilde{\theta}_B | \tilde{\theta}_A, z_1, \ldots, z_n\right] &= \mathbb{E}\left[\tilde{\theta}_B | z_1, \ldots, z_n\right] + (\tilde{\theta}_A - \mathbb{E}\left[\tilde{\theta}_A | z_1, \ldots, z_n\right])\gamma_n, \\
&= q_n + (\tilde{\theta}_A - q_n)\gamma_n, \qquad where \\
\gamma_n &= \frac{Cov(\tilde{\theta}_A, \tilde{\theta}_B | z_1, \ldots, z_n)}{Var(\tilde{\theta}_A | z_1, \ldots, z_n)} = \frac{\Sigma_n - \Sigma_0}{\Sigma_n}.
\end{aligned} \tag{A.6}$$

The third parameter r_n is used for updating $\widehat{e\theta}_{B,n}$, the belief on the equilibrium path (see

Eq. 10) using z_n^A (Equation 9):

$$
\begin{aligned}
\widehat{e\theta}_{B,n} &:= \mathbb{E}\left[\tilde{\theta}_B|\tilde{\theta}_A, z_1^A, \ldots, z_n^A\right], \\
&= \widehat{e\theta}_{B,n-1} + r_n z_n^A, \qquad e\theta_{B,0} = 0, \qquad where \\
r_n &= \frac{Cov(z_n^A, \tilde{\theta}_B|z_1^A, \ldots, z_{n-1}^A)}{Var(z_n^A|z_1^A, \ldots, z_{n-1}^A)}, \\
r_n &= \frac{(\alpha_n + \beta_n\gamma_{n-1})\Sigma_0(\Sigma_0 - 2\Sigma_{n-1})}{\sigma_e^2 + 2(\alpha_n + \beta_n\gamma_{n-1})^2\Sigma_0(\Sigma_0 - 2\Sigma_{n-1}) - \sigma_e^2\Sigma_{n-1}}.
\end{aligned}
\tag{A.7}
$$

A.1.2 Optimization trader A We first need the following lemma:

Lemma 4. *Fix the trades of B, $x_{B,n}$, by strategy (4), and consider the constants $\{\delta_n, \gamma_n, r_n\}$ defined in (A.5)-(A.7) and the conditional variance Σ_n in (A.3). Define the variable z_n^A as the unexpected component of the order flow y_n to trader A,*

$$
\begin{aligned}
z_n^A &:= y_n - \mathbb{E}\left[y_n|\tilde{\theta}_A, y_1, \ldots, y_{n-1}\right], \\
&= y_n - x_{A,n} - \alpha_n(\widehat{e\theta}_{B,n-1} - p_{n-1}) - \beta_n(q_{n-1} + \gamma_{n-1}(\widehat{e\theta}_{B,n-1} - q_{n-1}) - p_{n-1}).
\end{aligned}
\tag{A.8}
$$

Here, z_n^A is independent of $(\tilde{\theta}_A, y_1, \ldots, y_{n-1})$, and we have the properties

$$
\begin{aligned}
\sigma(\tilde{\theta}_A, z_1^A, \ldots, z_n^A) &= \sigma(\tilde{\theta}_A, \tilde{z}_1^A, \ldots, \tilde{z}_n^A) \\
\widehat{e\theta}_{B,n} - e\theta_{B,n} \in \sigma(\tilde{\theta}_A, y_1, \ldots, y_n) &= \sigma(\tilde{\theta}_A, z_1^A, \ldots, z_n^A), \qquad n = 1, \ldots, N.
\end{aligned}
\tag{A.9}
$$

Further, the three state variables,

$$
Y_n^{(1)} = \tilde{\theta}_A - p_n, \qquad Y_n^{(2)} = \widehat{e\theta}_{B,n} - p_n, \qquad Y_n^{(3)} = \widehat{e\theta}_{B,n} - q_n. \tag{A.10}
$$

have Markovian dynamics:

$$
\begin{aligned}
Y_n^{(1)} &= Y_{n-1}^{(1)} - \lambda(x_{A,n} + z_n^A + (\alpha_n + \beta_n)Y_{n-1}^{(2)} + \beta_n(\gamma_{n-1} - 1)Y_{n-1}^{(3)}), \\
Y_n^{(2)} &= Y_{n-1}^{(1)} + r_n z_n^A - \lambda(x_{A,n} + z_n^A + (\alpha_n + \beta_n)Y_{n-1}^{(2)} + \beta_n(\gamma_{n-1} - 1)Y_{n-1}^{(3)}), \\
Y_n^{(3)} &= Y_{n-1}^{(2)} + r_n z_n^A - \delta_n\left(x_{A,n} + z_n^A - (\alpha_n + \beta_n)Y_{n-1}^{(2)} + (2\alpha_n + \beta_n(1 + \gamma_{n-1}))Y_{n-1}^{(3)}\right).
\end{aligned}
\tag{A.11}
$$

The value function in any period n is quadratic in the trade $x_{A,n}$, and further the optimal trade $x^*_{A,n}$ is linear in the state variables $Y^{(1)}_{n-1}, Y^{(2)}_{n-1}, Y^{(3)}_{n-1}$ with coefficients given by Proposition 1 and A.18, such that the value function is also quadratic in the state variables $Y^{(1)}_{n-1}, Y^{(2)}_{n-1}, Y^{(3)}_{n-1}$. The coefficients of the optimal strategy (α_n and β_n) are obtained recursively.

Proof. The independence of z^A_n with respect to $(\tilde{\theta}_A, y_1, \ldots, y_{n-1})$ follows from the first line in Equation A.8 and the normality of all random variables.

Learning on and off the equilibrium path: $\sigma(\tilde{\theta}_A, z^A_1, \ldots, z^A_n) = \sigma(\tilde{\theta}_A, \tilde{z}^A_1, \ldots, \tilde{z}^A_n)$**.**

The result implies that the information set of trader A does not change when he has deviated in the past from the optimal strategy. This is proved by demonstrating that the process z^A_n in (A.8) is unaffected by past deviations.

In any round k, suppose trader A deviates from the optimal order with a random quantity ζ_k, and submits $x'_{A,k} = x^*_{A,k} + \zeta_k$. We need to show that this deviations does not affect z^A_i $\forall i \geq k$. We first observe directly that this deviation does not affect $z_{A,k}$ defined in (A.8): the impact on y_k is offset by the change in $x_{A,k}$. The deviation ζ_k increases y_k, and affects expected type q_k and price p_k (see 5 and 2). In round $k+1$, the affected q_k and p_k in turn imply changes in $x_{A,k+1}$ and $x_{B,k+1}$ according to (4), and subsequently changes in y_{k+1}. However, $z_{A,k+1}$ in (A.8) remains unaffected: the impact of ζ_k on y_{k+1} (first term) is exactly offset by the changes in the latter terms.

We conclude that past deviations by A changes the state variables and also $x_{B,n}$, but these changes are fully predictable because B follows a strategy linear in the state variables and his type (see (4)). The result is that the process z^A_n is unaffected. Accordingly, A can perfectly observe $\widehat{e\theta}_{B,n-1} - e\theta_{B,n-1}$, i.e., the difference of his expectation of $\tilde{\theta}_B$ on the equilibrium path (updating using z^A_n) and off the equilibrium path (updating using y_n).

This means that all the three state variables are observable.

Markovian dynamics of the state variables:

The state variables take the following recursive form:

$$\begin{aligned}
Y_n^{(1)} &:= \tilde{\theta}_A - p_n, \\
&= Y_{n-1}^{(1)} - \lambda y_n, \\
&= Y_{n-1}^{(1)} - \lambda(y_n + z_n^A - z_n^A), \\
&= Y_{n-1}^{(1)} - \lambda(x_{A,n} + z_n^A + (\alpha_n + \beta_n)Y_{n-1}^{(2)} + \beta_n(\gamma_{n-1} - 1)Y_{n-1}^{(3)}).
\end{aligned} \tag{A.12}$$

In the second line we insert the price equation, and in the last line we use the definition of z_n^A from Equation 9 to insert $Y_n^{(2)}$ and $Y_n^{(3)}$. We replace y_n by the state variables and the surprise component of the order flow z_n^A, which simplifies the recursive iterations since $\mathbb{E}\left[z_n^A | Y_{n-1}^{(1)}, Y_{n-1}^{(2)}, Y_{n-1}^{(3)}\right] = 0$. For the second state variable we have

$$\begin{aligned}
Y_n^{(2)} &:= \widehat{e\theta}_{B,n} - p_n, \\
&= Y_{n-1}^{(2)} + r_n z_n^A - \lambda y_n, \\
&= Y_{n-1}^{(1)} + r_n z_n^A - \lambda(x_{A,n} + z_n^A + (\alpha_n + \beta_n)Y_{n-1}^{(2)} + \beta_n(\gamma_{n-1} - 1)Y_{n-1}^{(3)}).
\end{aligned}$$

In the second line, we replace $\widehat{e\theta}_{B,n}$ by $Y_{n-1}^{(2)}$ using Equation 10, and insert the price equation as in A.12. The recursion for $Y_n^{(3)}$ becomes

$$\begin{aligned}
Y_n^{(3)} &:= \widehat{e\theta}_{B,n} - q_n, \\
&= Y_{n-1}^{(2)} + r_n z_n^A - \delta_n z_n, \\
&= Y_{n-1}^{(2)} + r_n z_n^A - \delta_n \left(x_{A,n} + z_n^A - (\alpha_n + \beta_n)Y_{n-1}^{(2)} + (2\alpha_n + \beta_n(1 + \gamma_{n-1}))Y_{n-1}^{(3)}\right).
\end{aligned} \tag{A.13}$$

In the second line we use the definition of q_n, which depends on z_n (the unexpected order flow from the market makers point of view). In the last line we replace z_n by a part predictable to A and the innovation z_n^A.

Quadratic value function:

This statement is proved using an induction argument. Suppose $Value_{n+1}^A$ is quadratic in $x_{A,n+1}$. Since the optimal trade $x_{A,n+1}^*$ is linear in the state variables $Y_n^{(1)}, Y_n^{(2)}, Y_n^{(3)}$ (see (15)), inserting $x^*_{A,n+1}$ into $Value^A_{n+1}$ yields a function quadratic in the state variables. Then,

$$Value^A_n \quad = \quad \mathbb{E}\left[x_{A,n}Y^{(1)}_n + Value^A_{n+1}|Y^{(1)}_{n-1}, Y^{(2)}_{n-1}, Y^{(3)}_{n-1}\right]. \tag{A.14}$$

The first term is the new trade, equal to $x_{A,n}(\tilde{\theta}_A - p_n)$, which is quadratic in $x_{A,n}$ since the expected price p_n depends linearly on $x_{A,n}$. The second term is the period n expectation of $Value^A_{n+1}$, which is also quadratic in $x_{A,n}$ since the three state variables iterated to period $n-1$ each depend linearly on $x_{A,n}$ (see the Markovian dynamics in (A.12)-(A.13)). Finally, the value function in period N is quadratic, as we see in (14), such that, by induction, the function is quadratic in all periods $N-1, \dots, 1$.

As the value function is quadratic, the optimal trade $x^*_{A,n}$ is obtained with a standard first-order condition. Verification of the second order condition confirms that we have a maximum.

Recursive strategy coefficients: The coefficients $\{b_{1,n}, b_{2,n}, b_{3,n}\}^N_{n=1}$ represent the aggressiveness with which A trades on the three state variables. These are obtained as follows. In period N, the value function is

$$\begin{aligned} Value^A_N =& \mathbb{E}\left[x_{A,N}Y^{(1)}_N|\tilde{\theta}_A, y_1, \dots, y_{N-1}\right], \\ =& \mathbb{E}\Big[x_{A,N}(Y^{(1)}_{N-1} - \lambda\left(z^A_N + x_{A,N} + (\alpha_N + \beta_N)Y^{(2)}_{N-1} + \beta_N(\gamma_{N-1} - 1)Y^{(3)}_{N-1}\right) \\ & |Y^{(1)}_{N-1}, Y^{(2)}_{N-1}, Y^{(3)}_{N-1}\Big], \\ =& x_{A,N}(\lambda x_{A,N} + (Y^{(1)}_{N-1} + (\alpha_N + \beta_N)Y^{(2)}_{N-1} + \beta_N(\gamma_{N-1} - 1)Y^{(3)}_{N-1}). \end{aligned} \tag{A.15}$$

In line 2 we fill in the Markovian recursion of $Y^{(1)}_N$ (A.12), and substitute in the unexpected order flow z^A_N (Eq. 9). The first order condition identifies the optimal trade $x^*_{A,N}$, and accordingly the constants $\{b_{1,N}, b_{2,N}, b_{3,N}\}$:

$$\begin{aligned} x^*_{A,N} =& \frac{Y^{(1)}_{N-1} - \lambda\left((\alpha_N + \beta_N)Y^{(2)}_{N-1} + \beta_N(\gamma_{N-1} - 1)Y^{(3)}_{N-1}\right)}{2\lambda} \\ =& b_{1,N}Y^{(1)}_{N-1} + b_{2,N}Y^{(2)}_{N-1} + b_{3,N}Y^{(3)}_{N-1} \end{aligned} \tag{A.16}$$

For period $N-1$, we have

$$Value^A_{N-1} \quad = \quad \mathbb{E}\left[x_{A,N-1}Y^{(1)}_{N-1} + Value^A_N | Y^{(1)}_{N-2}, Y^{(2)}_{N-2}, Y^{(3)}_{N-2}\right]. \tag{A.17}$$

We first fill in optimal trade $x^*_{A,N}$ (A.16), iterate the state variables back to period $N-2$ using (A.12), and again impose the linear structure of A.16. We now obtain $\{b_{1,N-1}, b_{2,N-1}, b_{3,N-1}\}$ as a function of $\{b_{1,N}, b_{2,N}, b_{3,N}\}$ and parameters $\alpha_{N-1}, \beta_{N-1}, \gamma_{N-2}, \delta_{N-1}$.

The recursive nature of the system is such that we can identify them for any step n:

$$\begin{aligned}
b_{1,n} =&(\lambda b_{0,n})^{-1}\Big(1 - b_{3,n+1}\delta_n + \lambda(-b_{2,n+1} + b_{1,n+1}\cdot \\
&(-2 + \lambda(2b_{1,n+1} + 2b_{2,n+1} + \alpha_{n+1} + \beta_{n+1}) + 2b_{3,n+1}\delta_n + \beta_{n+1}(-1+\gamma_n)\delta_n)\Big), \\
b_{2,n} =&(b_{0,n})^{-1}\Bigg(\lambda b_{1,n+1}(2\alpha_n + \alpha_{n+1} + 2\beta_n + \beta_{n+1}) - \\
&(\alpha_n + \beta_n + 2\lambda b^2_{2,n+1}(-1 + \lambda(\alpha_n + \beta_n)) + 2\lambda^2 b_{1,n+1}(\alpha_n + \beta_n)(b_{1,n+1} + \alpha_{n+1} + \beta_{n+1})) + \\
&b_{2,n+1}\Big(-1 - 2\lambda^2(\alpha_n + \beta_n)(2b_{1,n+1} + \alpha_{n+1} + \beta_{n+1}) + \\
&b_{3,n+1}\delta_n(\alpha_{n+1} + \beta_{n+1} + 2(\alpha_n + \beta_n)(b_{3,n+1} + \beta_{n+1}(-1+\gamma_n))\delta_n) + \\
&2\lambda(b_{1,n+1} + \alpha_n + \alpha_{n+1} + \beta_n + \beta_{n+1}) + 2b_{3,n+1}\delta_n + \beta_{n+1}(-1+\gamma_n)\delta_n\Big)\Bigg), \\
b_{3,n} =&(b_{0,n})^{-1}\Big((\lambda(b_{1,n+1} + b_{2,n+1})\beta_{n+1}(1-\gamma_n)(-1 + 2\alpha_n\delta_n) + 2b^2_{3,n+1}\delta_n(1 - 2\alpha_n\delta_n - \beta_n(1+\gamma_n)\delta_n)) \\
&+ \beta_n(1 - \gamma_n - 2\lambda(b_{1,n+1} + b_{2,n+1})(1-\gamma_n)(1 - \lambda(b_{1,n+1} + b_{2,n+1} + \alpha_{n+1} + \beta_{n+1})) - \beta_{n+1}\gamma_n\delta_n) \\
&- b_{3,n+1}(1 - 2\alpha_n\delta_n + \lambda(2b_{1,n+1} + 2b_{2,n+1} + \alpha_{n+1} + \beta_{n+1})(-1 + 2(\alpha_n + \beta_n\gamma_n)\delta_n)) + \\
&2b_{3,n+1}\delta_n(\beta_n\gamma_n + \beta_{n+1}(1-\gamma_n)(-1 + (2\alpha_n + \beta_n + \beta_n\gamma_n)\delta_n))\Big)
\end{aligned} \tag{A.18}$$

where the constant $b_{0,n}$ is defined as:

$$\begin{aligned}
b_{0,n} = 2\Big(&1 + \lambda(b_{1,n+1} + b_{2,n+1})(-1 + \lambda(b_{1,n+1} + b_{2,n+1} + \alpha_{n+1} + \beta_{n+1})) - \\
&b_{3,n+1}\delta_n + \lambda(b_{3,n+1}(\alpha_{n+1} + \beta_{n+1}) + b_{1,n+1}(2b_{3,n+1} - \beta_{n+1}(1-\gamma_n))) + \\
&\lambda(b_{2,n+1}(2b_{3,n+1} - \beta_{n+1}(1-\gamma_n)))\delta_n + b_{3,n+1}(b_{3,n+1} - \beta_{n+1}(1-\gamma_n))\delta^2_n\Big).
\end{aligned} \tag{A.19}$$

These parameters are general, in the sense that they are optimal strategies even if trader A deviated in the past (i.e., played off the equilibrium path).

The second order condition in period $n-1$ is:

$$2\lambda\left(1+(\lambda(b_{1,n}+b_{2,n})+b_{3,n}\delta_n)(-1+\lambda(b_{1,n}+b_{2,n}+\alpha_n+\beta_n)+(b_{3,n}-\beta_n(1-\gamma_{n-1})\delta_n)\right)\right)<0,$$
$$n=1,\ldots,N. \tag{A.20}$$

■

A.2 Algorithm to obtain numerical solution

The exogenous model parameters are N, σ_e, Σ_0, and λ. The key challenge is that the Baysian parameters (δ, r, γ) and Σ_n equations are given by forward recursions, while the optimal strategies (α, β and constants b_1, b_2, b_3) by backward recursions. To find a fixed point where all values align in both forward and backward recursion, we need to first "guess" a value for Σ_N, solve the model backwards from period N to 0, and confirm that the resulting Σ_0 from the Σ-recursion corresponds with the exogenously chosen Σ_0. If the two deviate, we need to change our guess of Σ_N and repeat the exercise until convergence is sufficiently close. Fortunately, the relation between the initial guess for Σ_N and the implied Σ_0 is monotonous, given the signs of the parameters in the Σ recursion (A.3).

In detail, we use the following program. In period N we solve numerically the system of equations for $\Sigma_{N-1}, \gamma_{N-1}, \gamma_N, \alpha_N, \beta_N, \delta_N$, and r_N using their respective recursions (the constants $b_{1,N}, b_{2,N}, b_{3,N}$ are zero). We also impose on the solution the second order condition A.20, and $\delta_N > 0$,$\gamma_{N-1} < 0$ as constraints. At time N this system has three roots, and in other periods it has five roots. If the initial guess of Σ_N is too far off, this may yield an empty set in which case we change our initial guess and restart the program. We repeat this step for all other periods including the equations for the constants b_1, b_2, b_3. In time 0, we compare the implied Σ_0 from the recursion to the exogenously chosen Σ_0.

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Table 1 Summary statistics

This table presents summary statistics for parent orders executed in Finnish stocks from January 1, 2007 to December 31, 2007. *Order size* is the total dollar volume of the parent order in USD thousands (Euros converted to USD at the sample average exchange rate). *Duration* is the length of the parent order in number of trading days. *Market order %* is the fraction of the parent order executed through market orders, instead of limit orders. *Directionality* is the absolute value of the difference between buying and selling dollar volumes, divided by the total parent order volume. *Participation rate* is the dollar volume of the parent order, divided by the total dollar volume traded in the market (single counted) over the lifetime of the order. *Same sign* (*Opposite sign*) is the total dollar volume of other parent orders trading in the same (opposite) direction as the parent order, divided by the total dollar volume traded in the market over the lifetime of the parent order. *Implementation shortfall* is the difference in basis points between the average execution price and the price prevailing at start of the parent order, and it is multiplied by minus one for sell orders.

	Mean	Std.Dev.	Q1	Median	Q3
Panel A: All parent orders (n=30,092)					
Order size ($1000)	954	6,683	81.8	196	779
Number of child trades	34.6	110	5.00	12.0	32.0
Duration (trading days)	0.44	0.65	0.032	0.27	0.67
Market order (%)	40.0	31.0	14.3	34.1	60.0
Directionality (based on $ volume)	1.00	0.02	1.00	1.00	1.00
Participation rate (%)	33.7	30.2	8.41	23.6	53.3
Same sign (%)	8.21	14.6	0.00	0.00	10.6
Opposite sign (%)	17.3	25.4	0.00	3.54	26.2
Implementation shortfall (bps)	6.03	106	-23.8	-1.09	29.6
Panel B: Buys only (n=14,694)					
Order size ($1000)	1,096	9,245	82.0	198	791
Number of child trades	36.6	139	5.00	12.0	33.0
Duration (trading days)	0.46	0.74	0.039	0.29	0.69
Market order (%)	41.0	31.0	16.7	37.5	61.5
Directionality (based on $ volume)	1.00	0.02	1.00	1.00	1.00
Participation rate (%)	33.4	30.1	8.23	23.3	52.7
Same sign (%)	8.49	14.8	0.00	0.00	11.4
Opposite sign (%)	17.2	25.4	0.00	3.45	26.5
Implementation shortfall (bps)	6.07	113	-24.4	0.00	30.1
Panel C: Sells only (n=15,398)					
Order size ($1000)	819	2,384	81.5	195	771
Number of child trades	32.6	72.1	5.00	12.0	32.0
Duration (trading days)	0.42	0.55	0.026	0.24	0.65
Market order (%)	38.0	31.0	12.1	33.3	58.3
Directionality (based on $ volume)	1.00	0.02	1.00	1.00	1.00
Participation rate (%)	33.9	30.2	8.61	23.8	53.8
Same sign (%)	7.94	14.5	0.00	0.00	9.86
Opposite sign (%)	17.3	25.4	0.00	3.67	25.9
Implementation shortfall (bps)	6.00	99.2	-23.0	-1.85	29.3

Table 2 Overlapping orders in same and opposite direction

This table presents the frequency of overlapping parent orders executed in Finish stocks from January 1, 2007 to December 31, 2007. For each new parent order, in the 30 minutes interval prior to its start we find the total volume of child trades executed from existing parent orders. Next, we rank these existing volumes into terciles based on whether the existing parent orders are trading in the same direction (same sign) and in opposite direction (opposite sign) to the new incoming parent order. We add a category 0 if no volume is executed in the same or opposite direction. The table shows the frequency of new parent orders for each same sign volume rank and opposite sign volume rank combination. We report the values as a percentage of total orders in parentheses ($N = 30,092$).

		Volume rank (opposite sign)			
		0	1	2	3
Volume rank (same sign)	0	13,918	2,633	2,311	1,883
		(46.3)	(8.8)	(7.7)	(6.3)
	1	1,888	627	414	191
		(6.3)	(2.1)	(1.4)	(0.6)
	2	1,558	298	686	462
		(5.2)	(1.0)	(2.3)	(1.5)
	3	1,594	149	376	1,104
		(5.3)	(0.5)	(1.3)	(3.7)

Table 3 The impact of existing orders on the new orders' sign and size

For the logit regressions, the dependent variable is equal to 1 for a new parent buy order and 0 for a new parent sell order. We present the marginal effects based on the means of the independent variables. For the OLS regressions, the dependent variable is the log volume of the new parent order executed during the first 30 minutes of the new order. All independent variables are calculated based on the 30 minute interval prior to commence of the new order. *Active parent buys* (*sells*) is the count of the number of existing parent buy (sell) orders. *Ln(Active buy (sell) volume)* is the logarithm of 1 + dollar buy (sell) volume from existing parent orders. *Return* is the stock return and *$ Volume* is the total dollar volume transacted in the stock. *Volatility* is the log of the high price minus the log of the low price. We include only parent orders lasting over 30 minutes. All regressions control for stock fixed effects. t-statistics clustered by stock are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Logit (Parent buy order)		OLS			
			Buy volume		Sell volume	
	1	2	3	4	5	6
Active parent buys (#)	-0.014		0.008		0.183***	
	(-0.72)		(0.33)		(4.81)	
Active parent sells (#)	0.041*		0.224***		0.057*	
	(1.73)		(7.65)		(1.97)	
Ln(Active buy volume)		-0.008***		0.005		0.034***
		(-3.62)		(1.04)		(5.34)
Ln(Active sell volume)		0.007***		0.041***		0.007
		(2.67)		(6.80)		(1.30)
Return	-2.575***	-2.520***	2.097	2.249	1.484	1.417
	(-4.51)	(-4.42)	(1.21)	(1.35)	(1.17)	(1.11)
Ln($ Volume)	0.001	0.004	0.114***	0.101***	0.117***	0.109***
	(0.16)	(1.04)	(7.78)	(6.76)	(9.12)	(7.87)
Volatility	-0.211	-0.259	-1.158	-1.354	1.692	1.385
	(-0.54)	(-0.66)	(-0.79)	(-0.99)	(1.44)	(1.11)
Constant			5.225***	5.367***	1.268***	1.290***
			(35.84)	(35.96)	(9.94)	(10.24)
Observations	18,597	18,597	9,267	9,267	9,322	9,322
Adjusted/Pseudo R-squared	0.015	0.017	0.236	0.235	0.285	0.283

Table 4 The correlation between the realized spread and institutional same sign and opposite sign flow.

The dependent variable is the average realized spread which proxies for price reversals (see Equation 18), and we calculate it based on the midpoint price 5, 10, or 20 trades from the time of the original trade. We average the realized spread (in basis points) over all trades on Nasdaq during the lifetime of the parent order, and winsorize at the 1% level. *Same sign %* (*Opposite sign %*) is the total dollar volume of other parent orders trading in the same (opposite) direction as the parent order, divided by the total dollar volume traded in the market over the lifetime of the parent order, expressed as a percentage. *Participation rate %* is the total dollar volume of the parent order, divided by the total dollar volume traded in the market over the lifetime of the parent order, expressed as a percentage. *$ order size* is the total dollar volume of the parent order in USD thousands. *Trade size* is the average trade size of the child trades in the parent order in number of shares. *Volatility* is the standard deviation of 30 minute stock returns in basis points. *Market order %* is the number of child trades executed through market orders as a percentage of all child trades in the parent order. *Duration* is the length of the parent order in number of trading days. *No counterparty* is an indicator variable equal to 1 if there are no other parent orders executing during the duration of the parent order. All regressions control for stock fixed effects. t-statistics clustered by stock are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

	Realized spread		
	5	10	20
Same sign %	-0.037**	-0.080***	-0.133***
	(-2.38)	(-3.62)	(-3.28)
Opposite sign %	0.033***	0.092***	0.183***
	(2.87)	(5.18)	(5.31)
Participation rate %	0.079	0.117	-0.033
	(1.53)	(1.23)	(-0.20)
Ln($ order size)	0.617***	-0.135	-0.873*
	(2.81)	(-0.38)	(-1.85)
Ln(Trade size)	-1.147***	-0.748**	0.376
	(-4.11)	(-2.17)	(0.74)
Volatility	-0.010	-0.008	0.010
	(-1.20)	(-0.65)	(0.48)
Market order %	0.348***	0.302***	0.229***
	(10.32)	(9.68)	(8.31)
Duration	-0.565**	-0.365	-0.448
	(-2.53)	(-1.23)	(-1.11)
No counterparty	-0.159	-0.724	-1.652
	(-0.39)	(-1.02)	(-1.65)
Observations	19,101	19,101	19,101
R-squared	0.225	0.105	0.055

Table 5 Order detection in real time

The dependent variable is *Participation rate %*, *Same sign %*, *Opposite sign %* or *Net opposite sign %*, all measured at 30 minute intervals. *Participation rate %* is the dollar volume of the parent order executed in the 30 minute interval t, divided by the market dollar volume and expressed as a percentage. *Same sign %* (*Opposite sign %*) is the total dollar volume of other parent orders trading in the same (opposite) direction as the parent order, divided by the total dollar volume traded in the market during t, and expressed as a percentage. *Net opposite sign %* is the difference between *Opposite sign %* and *Same sign %*. All other variables are defined in Table 4.

	Participation rate %	Same sign %	Opposite sign %	Net opposite sign %
Panel A: Lagged regressions				
Realized spread$_{t-1}$	0.044***	-0.003	0.029***	0.031***
	(4.29)	(-0.51)	(2.91)	(3.78)
Same sign %$_{t-1}$	0.002	0.445***	0.023***	-0.422***
	(0.30)	(36.72)	(3.89)	(-31.00)
Opposite sign %$_{t-1}$	0.023***	0.011***	0.467***	0.456***
	(4.90)	(3.32)	(39.58)	(38.77)
Participation rate %$_{t-1}$	0.331***	-0.000	-0.011***	-0.010**
	(28.85)	(-0.05)	(-2.68)	(-2.01)
Order imbalance$_{t-1}$	2.878***	1.234***	1.353***	0.119
	(6.41)	(4.35)	(5.25)	(0.40)
Volatility$_{t-1}$	-0.064	-0.090	-0.005	0.086
	(-0.57)	(-1.08)	(-0.04)	(0.57)
Return$_{t-1}$	-0.422**	0.029	-0.424**	-0.453**
	(-2.08)	(0.22)	(-2.27)	(-2.55)
Ln(Volume)$_{t-1}$	-1.828***	-0.445***	-1.032***	-0.587***
	(-14.09)	(-3.86)	(-9.72)	(-4.81)
Constant	27.175***	8.117***	15.187***	7.070***
	(18.28)	(6.58)	(11.53)	(5.59)
Observations	150,991	150,991	150,991	150,991
R-squared	0.174	0.206	0.251	0.205
Panel B: Contemporaneous regressions				
Realized spread$_t$	0.135***	0.009	0.056***	0.047***
	(8.95)	(1.06)	(4.05)	(5.17)
Order imbalance$_t$	13.621***	4.612***	11.007***	6.395***
	(11.26)	(12.83)	(10.63)	(7.39)
Volatility$_t$	-0.323	-0.153	-0.273	-0.120
	(-1.31)	(-0.72)	(-0.83)	(-0.48)
Return$_t$	1.699***	0.390**	0.783***	0.393**
	(5.12)	(2.42)	(2.85)	(2.01)
Ln(Volume)$_t$	-3.117***	-0.327	-1.174***	-0.847***
	(-12.12)	(-1.18)	(-4.93)	(-3.83)
Constant	42.721***	8.928***	18.130***	9.202***
	(16.68)	(3.68)	(7.59)	(4.50)
Observations	176,134	176,134	176,134	176,134
R-squared	0.106	0.008	0.037	0.011

Panel C: Summary statistics of dependent and main independent variables

	Mean	Std.Dev.
Participation rate (%)	21.90	31.39
Same sign (%)	6.835	17.93
Opposite sign (%)	11.67	25.39
Realized spread	9.804	26.67

Table 6 Implementation shortfall, price impact and same sign and opposite sign institutional trading

The dependent variables are *Implementation shortfall*, overall price impact (*OPI*), permanent price impact (*PPI*), and temporary price impact (*TPI*). *Implementation shortfall* is the difference in basis points between the average execution price and the price prevailing at start of the parent order, and it is multiplied by minus one for sell orders. *OPI* is the market adjusted stock return based on mid-quote prices from the beginning of the parent order to the end of the parent order, in basis points. *PPI* is the market adjusted stock return based on mid-quote prices from the beginning of the parent order to the mid-quote price 24 hours after the end of the parent order, in basis points. *TPI* is the difference between *OPI* and *PPI*. *Implementation shortfall*, *OPI*, *PPI* and *TPI* are winsorized at the 1% level. *Same sign %* (*Opposite sign %*) is the total dollar volume of other parent orders trading in the same (opposite) direction as the parent order, divided by the total dollar volume traded in the market over the duration of the parent order, expressed as a percentage. *Participation rate %* is the total dollar volume of the parent order, divided by the total dollar volume traded in the market over the duration of the parent order, expressed as a percentage. The remaining variables are defined in Table 4. All regressions control for stock fixed effects. t-statistics clustered by stock are reported in parentheses. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

Panel A: Regression results				
	IS	OPI	PPI	TPI
Same sign %	0.357***	0.597***	0.806***	-0.210*
	(5.13)	(5.29)	(5.53)	(-1.94)
Opposite sign %	-0.300***	-0.426***	-0.606***	0.183**
	(-6.36)	(-6.06)	(-5.07)	(2.09)
Participation rate %	0.363***	0.442***	0.367***	0.076
	(7.06)	(4.96)	(3.19)	(1.00)
Ln($ order size)	18.934***	27.596***	26.359***	1.190
	(11.70)	(9.91)	(7.91)	(0.64)
Ln(Trade size)	-15.619***	-21.964***	-16.340***	-5.604***
	(-11.00)	(-10.10)	(-5.91)	(-2.84)
Volatility	0.176***	0.174***	0.062	0.112*
	(6.13)	(3.49)	(1.07)	(1.95)
Market order %	71.632***	0.341***	0.442***	-0.102*
	(11.17)	(6.33)	(5.08)	(-1.90)
Duration	-9.169***	-14.457***	-10.153**	-4.275*
	(-2.71)	(-3.18)	(-1.99)	(-1.83)
No counterparty	1.460	4.466	1.936	2.471
	(0.69)	(1.63)	(0.46)	(0.70)
Observations	20,708	20,348	20,352	20,348
R-squared	0.093	0.055	0.023	0.002

Panel B: Summary statistics of price impact measures		
	Mean	Std.Dev.
OPI (bps)	4.631	126.0
PPI (bps)	4.665	233.5
TPI (bps)	-0.086	200.1